Daily Practice Problems for ECON 205

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July 4, 2020

These problems are meant to be similar to problems given in lecture. They are meant to help work out the kinks before trying more difficult problems on the problem set.

Lecture 1

1. Consider the following model. Describe its agents, their objective, the exogenous and endogenous variables, and any assumptions. (You do not have to solve the model or write down any equations.)

   Anthony wants to pay off his student debt as fast as he can. He owes money to three banks. At bank 1, he owes amount B_1 at interest rate r_1. Similarly, he owes B_2 at rate r_2 at bank 2 and B_3 at r_3 at bank 3. He can spend \$1,000 per month repaying his loans. How should he divide it between the loans?

2. Draw indifference curves for a consumer who gets no utility from good y. Find a utility function that could represent the preferences.

3. Consider the preferences $U(x, y) = x^{1/2}y^{1/2}$.
   (a) Find the individual's marginal rate of substitution. Interpret.
   (b) Calculate the MRS at $(1, 3)$ and $(3, 1)$. Interpret.
   (c) The rates of substitution derived in part (b) are not the same. What property of preferences explains why? What is the economic justification?

Lecture 2

1. Suppose an individual has preferences given by $U(x, y) = x^{1/4}y^{3/4}$. Income is I and prices are p_x and p_y. (For simpler practice, you can use $I = 96$, $p_y = 48$, and $p_x = 16$.)
   (a) Find the individual's ordinary demand.
   (b) Find the indirect utility function.

(c) Find the expenditure function for some level of utility $\bar{u}$.

(d) Find the compensated demand function.

(e) *Bonus:* You could have solved for compensated demand in two ways. What is the other one? Verify that it gives the same answer.

2. Solving the ordinary demand problem gives the condition $MRS = p_x/p_y$. Interpret it.

Lecture 3

$$\begin{aligned} x^*(p_x, p_y, I) &= \frac{I}{4p_x}, \\ y^*(p_x, p_y, I) &= \frac{3I}{4p_y}, \\ x^c(p_x, p_y, \bar{u}) &= 3^{-3/4}\bar{u}\left(\frac{p_y}{p_x}\right)^{3/4}, \\ y^c(p_x, p_y, \bar{u}) &= 3^{1/4}\bar{u}\left(\frac{p_x}{p_y}\right)^{1/4}, \\ e(p_x, p_y, \bar{u}) &= \frac{4}{3^{3/4}}\bar{u}p_x^{1/4}p_y^{3/4}. \end{aligned}$$

1. Consider the functions above from the last set of practice problems.
   (a) Is good x normal or inferior?
   (b) Will CV or EV be larger for good x?
   (c) Initially, $I = 320$, $p_x = 16$, and $p_y = 48$. Then suppose the government places a tax on each unit of x sold, raising the price to $p_x + \tau = 20$. Break the effect of the tax on consumption of x into substitution and income effects.
   (d) Find the income tax that has the same effect on utility.
   (e) Calculate CV and EV for the income tax.

Lecture 4

1. Consider the tax from the lecture 3 practice problems. Income is $I = 320$, prices are $p_x = 16$ and $p_y = 48$, and the tax raises the price of x to $p_x + \tau = 20$. Recall that ordinary demand for x is $x^*(p_x, p_y, I) = \frac{I}{4p_x}$. What is deadweight loss from the tax?

2. A consumer has intertemporal utility function $U(c_0, c_1) = \ln(c_0) + \frac{2\ln(c_1)}{3}$. She earns $I_0 = 90$ in the current period and $I_1 = 110$ in the next period. The interest rate is $r = 1/10$.

(a) What is the consumer's budget constraint?

(b) What is the consumer's consumption in each period?

(c) Suppose that the consumer's income changes to $I_0 = 0$, $I_1 = 209$. What is the consumer's optimal consumption in each period?

Lectures 5 and 6

1. Suppose that Ms. Smith faces the following risky situation. With probability $\alpha = 0.5$ her wealth is $I_1 = 64$, and with probability $1 - \alpha = 0.5$ it is $I_2 = 100$. Suppose that Ms. Smith's Bernoulli utility function over wealth is $v(x) = \sqrt{x}$.

   (a) What is the expected value of Ms. Smith's wealth, $\bar{I}$? What is her expected utility, $U(64, 100; 0.5, 0.5)$?

   (b) What is Ms. Smith's certainty equivalent wealth level, x^{CE}? What is the value of her risk premium, R?

   (c) What is the price (per dollar of coverage) p of actuarially fair insurance in this case? If Ms. Smith could buy insurance at p per dollar of coverage, how much insurance would she buy?

Lectures 7 and 8

1. A firm's production function is $f(l, k) = l^{\alpha}k^{1-\alpha}$, where $0 < \alpha < 1$. It can hire labor in any quantity at wage w, but its capital stock is fixed at k_0. Each unit of capital was hired at rental rate v.

   (a) How much labor does the firm need to hire to produce q units of output?

   (b) What is the (short-run) cost of producing q units? What is the variable cost? Short-run marginal cost?

   (c) What is the firm's short-run supply function?

   (d) Why doesn't the firm's supply function depend on the cost of capital v?

2. Consider a firm in the long run with production function $f(l, k) = l^{1/5}k^{3/5}$.

   (a) What are conditional factor demands?

   (b) What is the firm's cost function?

   (c) What is the firm's supply function?

   (d) How would you find the firm's profit function and factor demand functions?

Lectures 9 and 10

1. Consider an industry where each firm has supply function $q^* = 10\sqrt{p}$ and market demand is $Q_D = 800/\sqrt{p}$. Assume that the number of firms is fixed at 5 in the short run and suppose that each firm's profit function is $10p^{3/2} - 30p$.
   (a) What is industry supply?
   (b) What is the short-run equilibrium (p^*, Q^*)?
   (c) What is each firm's profit in the short-run equilibrium?
   (d) Find the price, quantity, and number of firms in the market in the long-run equilibrium.

Lecture 11

1. George and Karla want to trade sources (x) and documents (y). George's preferences are given by $U_G(x, y) = 3\ln(x) = \ln(y)$; Karla's are given by $U_K(x, y) = \ln(x) + 3\ln(y)$. Each enters the market with 8 units of each good.
   (a) What is the value of each endowment?
   (b) What are ordinary demands when prices are $(p, 1)$?
   (c) What is the price p of x in terms of y in general equilibrium? What are the final bundles?