Daily Practice Problems for ECON 205
Solutions

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These problems are meant to be similar to problems given in lecture. They are meant to help work out the kinks before trying more difficult problems on the problem set.

Lecture 1

1. Consider the following model. Describe its agents, their objective, the exogenous and endogenous variables, and any assumptions. (You do not have to solve the model or write down any equations.)

   Anthony wants to pay off his student debt as fast as he can. He owes money to three banks. At bank 1, he owes amount B_1 at interest rate r_1. Similarly, he owes B_2 at rate r_2 at bank 2 and B_3 at r_3 at bank 3. He can spend \$1,000 per month repaying his loans. How should he divide it between the loans?

   Solution: Anthony is the only agent. His objective is to pay off his debt as fast as possible. Technically, this means that he wants to allocate his payments between the loans to minimize his total loan payments. (The banks could be seen as agents, but they do not make any choices—they have just given loans out in the past and now passively collect them.)

   The endogenous variables are the payments to each loan, which I call p_1, p_2, and p_3. The exogenous variables are his balances and interest rates at each bank and the amount that he can pay off each month.

   The main assumption is that Anthony spends some fixed amount each month repaying the debt. This is not the only reasonable assumption; the amount he can repay might vary each month.

   If the interest is compounded continuously and the best payment plan today is the same as the best payment plan tomorrow (e.g. we can ignore time), Anthony's problem could be written as

$$\min_{p_1,p_2,p_3} (B_1 - p_1)\exp(r_1) + (B_2 - p_2)\exp(r_2) + (B_3 - p_3)\exp(r_3).$$

Fun fact: Anthony should allocate all of his money to the loan with the highest interest rate.

2. Draw indifference curves for a consumer who gets no utility from good y. Find a utility function that could represent the preferences.

Solution: Indifference curves should be vertical lines (assuming that good y is on the y-axis). The reason is that adding good y, which involves moving upwards vertically, does not contribute to utility, so the individual stays on the same indifference curve.

The only requirement for the utility function is that y does not appear, because y does not contribute to utility. One possibility is $U(x, y) = x$.

3. Consider the preferences $U(x, y) = x^{1/2}y^{1/2}$.

(a) Find the individual's marginal rate of substitution. Interpret.

Solution: The MRS equals $-\frac{\partial U/\partial x}{\partial U/\partial y}$.

$$\begin{aligned} \frac{\partial U}{\partial x} &= \frac{1}{2}x^{-1/2}y^{1/2} \\ \frac{\partial U}{\partial y} &= \frac{1}{2}x^{1/2}y^{-1/2} \\ MRS &= -\frac{x^{-1/2}y^{1/2}}{x^{1/2}y^{-1/2}} = \frac{y}{x}. \end{aligned}$$

(b) Calculate the MRS at $(1, 3)$ and $(3, 1)$. Interpret.

Solution: At $(1, 3)$, the MRS is -3, so we say the individual is willing to give up 3 units of y for an additional unit of x. However, this is only true on a small scale. If the individual actually gave up 3 units of y, then he would have zero units of y and hence zero utility, leaving him worse off! What we really mean is that he would trade something like .03 units of y for .01 units of x—the change must be near the original bundle.

At $(3, 1)$, the MRS is $-1/3$, meaning that the individual is willing to trade $1/3$ of a unit of y for an additional unit of x, but again with the caveat that the trade would be on a small scale.

(c) The rates of substitution derived in part (b) are not the same. What property of preferences explains why? What is the economic justification?

Solution: The property is convexity, which causes the indifference curve to bow inwards. This causes the MRS to decline as x increases, which we saw by comparing $(1, 3)$ to $(3, 1)$ (both are on the same indifference curve).

The economic reason for convex preferences is that we (usually) prefer to have a mix of two goods because of diminishing marginal returns. If the individual already has a lot of y, then he much prefers to get a bit more x, and the MRS indicates that he would pay more in terms of y for it. If he has little y, then he prefers to get a bit more y, and is thus willing to trade less for x than before.

Lecture 2

1. Suppose an individual has preferences given by $U(x, y) = x^{1/4}y^{3/4}$. Income is I and prices are p_x and p_y. (For simpler practice, you can use $I = 96$, $p_y = 48$, and $p_x = 16$.)

(a) Find the individual's ordinary demand.

Solution: We want the bundle that maximizes utility given the income constraint. The Lagrangian is

$$\mathcal{L} = x^{1/4}y^{3/4} + \lambda(I - p_x x - p_y y).$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{1}{4}\left(\frac{y}{x}\right)^{3/4} - p_x\lambda = 0,$$
$$\frac{\partial \mathcal{L}}{\partial y} = \frac{3}{4}\left(\frac{x}{y}\right)^{1/4} - p_y\lambda = 0.$$

Substituting for λ gives $x^* = (\frac{1}{3})(\frac{p_y}{p_x})y^*$. Substituting into the resource constraint for y then gives

$$
\begin{aligned}
I &= \frac{p_y}{3}y^* + p_y y^* \\
y^*(p_x, p_y, I) &= \frac{3I}{4p_y}.
\end{aligned}
$$

Then $x^*(p_x, p_y, I) = \frac{I}{4p_x}$.

(b) Find the indirect utility function.

Solution: The indirect utility function is just the utility function evaluated at the ordinary demand bundle,

$$
\begin{aligned}
U(p_x, p_y, I) &= U(x^*(p_x, p_y, I), y^*(p_x, p_y, I)) \\
&= \left(\frac{I}{4p_x}\right)^{1/4}\left(\frac{3I}{4p_y}\right)^{3/4} \\
&= \frac{I}{4}p_x^{-1/4}p_y^{-3/4}3^{3/4}.
\end{aligned}
$$

(c) Find the expenditure function for some level of utility $\bar{u}$.

Solution: By duality, the income I that achieves utility $\bar{u}$ in the indirect utility function is the cheapest way to achieve utility $\bar{u}$. So if

$$
\bar{u} = V(p_x, p_y, I) = \frac{I}{4}p_x^{-1/4}p_y^{-3/4}3^{3/4},
$$

then we can find $e(p_x, p_y, \bar{u})$ by solving the above equation for I,

$$
\begin{aligned}
I &= \frac{4}{3^{3/4}}\bar{u}p_x^{1/4}p_y^{3/4} \\
e(p_x, p_y, \bar{u}) &= \frac{4}{3^{3/4}}\bar{u}p_x^{1/4}p_y^{3/4}.
\end{aligned}
$$

(d) Find the compensated demand function.

Solution: We could either use Shephard's lemma to go from expenditure to compensated demand, or we solve the expenditure minimization problem directly from the Lagrangian. I opt for Shephard's lemma:

$$\begin{aligned} x^c(p_x, p_y, \bar{u}) &= \frac{\partial e(p_x, p_y, \bar{u})}{\partial p_x} \\ &= 3^{-3/4}\bar{u}\left(\frac{p_y}{p_x}\right)^{3/4}, \\ y^c(p_x, p_y, \bar{u}) &= 3^{1/4}\bar{u}\left(\frac{p_x}{p_y}\right)^{1/4}. \end{aligned}$$

(e) *Bonus:* You could have solved for compensated demand in two ways. What is the other one? Verify that it gives the same answer.

Solution: You could have solved the expenditure minimization problem,

$$\min_{x,y} p_x x + p_y y \text{s.t.} U(x, y) = \bar{u}.$$

This can be done like the Langrangian in (a).

2. Solving the ordinary demand problem gives the condition $MRS = p_x/p_y$. Interpret it.

Solution: The MRS tells us how much good y an individual is willing to trade for one unit of good x. The price ratio tells us how much good y the consumer can trade for one unit of good x in the market: you can think of the consumer being able to sell one unit of x in the market for p_x, then spending it all on y to get p_x/p_y units of y.

The $MRS = p_x/p_y$ condition means that the consumer must be willing to trade x and y at the same rate as the market. If not, she would be able to sell x to buy y (or vice versa) at market rates and improve her utility.

Lecture 3

$$\begin{aligned}
x^*(p_x, p_y, I) &= \frac{I}{4p_x}, \\
y^*(p_x, p_y, I) &= \frac{3I}{4p_y}, \\
x^c(p_x, p_y, \bar{u}) &= 3^{-3/4}\bar{u}\left(\frac{p_y}{p_x}\right)^{3/4}, \\
y^c(p_x, p_y, \bar{u}) &= 3^{1/4}\bar{u}\left(\frac{p_x}{p_y}\right)^{1/4}, \\
e(p_x, p_y, \bar{u}) &= \frac{4}{3^{3/4}}\bar{u}p_x^{1/4}p_y^{3/4}.
\end{aligned}$$

1. Consider the functions above from the last set of practice problems.

   (a) Is good x normal or inferior?

   > **Solution:** To tell if it is normal or inferior, we need to know if demand for x increases or decreases when income rises. Since ordinary demand for x is increasing in I (more precisely, $\frac{\partial x^*}{\partial I} = (4p_x)^{-1} > 0$), it is normal.

   (b) Will CV or EV be larger for good x?

   > **Solution:** CV, because good x is normal. This is because $\frac{\partial x^*}{\partial I}$ has the same sign as $\frac{\partial x^c}{\partial u}$, so the fact that x is normal means that the compensated demand curve shifts to the right when utility increases. CV is the integral of the compensated demand curve at the higher utility. Because the compensated demand curve is further to the right at the higher utility, the area under the curve is larger and CV > EV. (See Figure 5 in the notes for an illustration.)

   (c) Initially, $I = 320$, $p_x = 16$, and $p_y = 48$. Then suppose the government places a tax on each unit of x sold, raising the price to $p_x + \tau = 20$. Break the effect of the tax on consumption of x into substitution and income effects.

   > **Solution:** The total effect (substitution plus income) is that consumption of x falls from $x^*(16, 48, 320) = 5$ to $x^*(20, 48, 320) = 4$.
   >
   > To find the substitution effect, we want to find the amount of x consumed to stay at the $(16, 48, 320)$ level of utility at the new prices, $p_x = 20$ and $p_y = 48$. Since the consumer also buys 5 units of y, initial utility is $5^{1/4}5^{3/4} = 5$. By

the compensated demand function, to reach utility 5 at the new prices would require $x^c(20, 48, 5) = 3^{-3/4} 5 \left(\frac{48}{20}\right)^{3/4} \approx 4.23$ units of x. The substitution effect is therefore $4.23 - 5 = -.77$. The income effect is the total change minus the substitution effect, $-1 + .77 = -.23$.

(d) Find the income tax that has the same effect on utility.

Solution: We need to find the consumer's utility after the tax and the income that achieves that utility at pre-tax prices. Post-tax utility is $U(4, 5) = 4^{1/4} 5^{3/4} \approx 4.73$.

The income needed to achieve that utility when prices are $p_x = 16$ and $p_y = 48$ is given by the expenditure function,

$$\frac{4}{3^{3/4}}(4.73)16^{1/4}48^{3/4} = 302.72.$$

The equivalent income tax is thus $320 - 302.72 = 17.28$.

(e) Calculate CV and EV for the income tax.

Solution: CV is the difference in expenditure to attain the higher, pre-tax utility level $u = 5$.

$$\begin{aligned} CV &= e(20, 48, 5) - e(16, 48, 5) \\ &= \left[\frac{4}{3^{3/4}}(5)20^{1/4}48^{3/4}\right] - 320 \\ &= 338.35 - 320 = 18.35. \end{aligned}$$

EV is the difference for the lower, post-tax level $u = 4.73$.

$$\begin{aligned} EV &= e(20, 48, 4.73) - e(16, 48, 4.73) \\ &= 320 - \left[\frac{4}{3^{3/4}}(4.73)16^{1/4}48^{3/4}\right] \\ &= 320 - 302.72 = 17.28. \end{aligned}$$

EV is the same as the equivalent income tax from part (d). This is because the equivalent income tax left the consumer with the post-tax level of utility.

Lecture 4

1. Consider the tax from the lecture 3 practice problems. Income is $I = 320$, prices are $p_x = 16$ and $p_y = 48$, and the tax raises the price of x to $p_x + \tau = 20$. Recall that ordinary demand for x is $x^*(p_x, p_y, I) = \frac{I}{4p_x}$. What is deadweight loss from the tax?

> **Solution:** Deadweight loss is the loss in total surplus. In this case, the only agents are the consumer and the government, so we must compare the consumer's surplus change ΔCS to the government's surplus change, given by tax revenue. After the tax, the consumer buys 4 units of x, so government revenue is 16.
>
> Consumer surplus is
>
> $$\begin{aligned}\Delta CS &= \int_{p_x}^{p_x+\tau} x^*(p_x, p_y, I)\, dp_x \\ &= \int_{16}^{20} \frac{320}{4p_x}\, dp_x \\ &= 80 \left(\ln(p_x)\Big|_{16}^{20} \right) = 80\,(\ln(20) - \ln(16)) = 17.85.\end{aligned}$$
>
> Deadweight loss is therefore $17.85 - 16 = 1.85$.

2. A consumer has intertemporal utility function $U(c_0, c_1) = \ln(c_0) + \frac{2\ln(c_1)}{3}$. She earns $I_0 = 90$ in the current period and $I_1 = 110$ in the next period. The interest rate is $r = 1/10$.

   (a) What is the consumer's budget constraint?

   > **Solution:** The present value of the consumer's spending cannot exceed the present value of her income. (You could also use the future values and get the same answer.)
   >
   > $$\begin{aligned} c_0 + \frac{1}{1+r} c_1 &= I_0 + \frac{1}{1+r} I_1 \\ c_0 + \frac{10}{11} c_1 &= 90 + 100 = 190 \end{aligned}$$

   (b) What is the consumer's consumption in each period?

Solution: The consumer maximizes utility subject to the resource constraint, just like in the ordinary demand problem.

$$\mathcal{L} = \ln(c_0) + \frac{2\ln(c_1)}{3} + \lambda\left(I_0 + \frac{10}{11}I_1 - c_0 - \frac{10}{11}c_1\right).$$

Taking first-order conditions and solving,

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_0} &= \frac{1}{c_0} - \lambda = 0 \\
\frac{\partial \mathcal{L}}{\partial c_0} &= \frac{2}{3c_1} - \lambda\frac{10}{11} = 0 \\
\frac{1}{c_0} &= \frac{22}{30c_1} \\
c_1 &= \frac{11}{15}c_0 \\
190 &= c_0 + \frac{10}{11}\frac{11}{15}c_0 = c_0 + \frac{2}{3}c_0 = \frac{5}{3}c_0 \\
c_0^* &= \frac{3}{5}(190) = 114 \\
c_1^* &= \frac{11}{15}(114) = 83.6.
\end{aligned}$$

(c) Suppose that the consumer's income changes to $I_0 = 0$, $I_1 = 209$. What is the consumer's optimal consumption in each period?

Solution: Optimal consumption in each period does not change. The reason is that the present value of the consumer's income is the same as before: 209 at $t = 1$ is worth $\frac{10}{11}209 = 190$ at $t = 0$. The distribution of income does not matter to the consumer because only the present value appears in the budget constraint. The economic interpretation is that the consumer can effortlessly move income across periods by borrowing and lending, so her total income is what's important, not its division between periods.

Lectures 5 and 6

1. Suppose that Ms. Smith faces the following risky situation. With probability $\alpha = 0.5$ her wealth is $I_1 = 64$, and with probability $1 - \alpha = 0.5$ it is $I_2 = 100$. Suppose that Ms. Smith's Bernoulli utility function over wealth is $v(x) = \sqrt{x}$.

   (a) What is the expected value of Ms. Smith's wealth, $\bar{I}$? What is her expected utility, $U(64, 100; 0.5, 0.5)$?

   Solution:

   $$\overline{I} = 82$$

   $$U(64, 100; \frac{1}{2}, \frac{1}{2}) = \frac{1}{2}\sqrt{60} + \frac{1}{2}\sqrt{100} = 9$$

   (b) What is Ms. Smith's certainty equivalent wealth level, x^{CE}? What is the value of her risk premium, R?

   Solution:

   $$x^{CE} = 81$$

   $$R = \overline{I} - x^{CE} = 1$$

   (c) What is the price (per dollar of coverage) p of actuarially fair insurance in this case? If Ms. Smith could buy insurance at p per dollar of coverage, how much insurance would she buy?

   Solution:

   $$p = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

   The budget constraint if the consumer buys insurance of $(x_1 - 64)$ will be

   $$x_2 = 100 - (x_1 - 64)$$

   The optimality condition requires

$$\frac{x_2}{x_1} = 1$$

This implies that

$$x_1^* = x_2^* = 82$$

Therefore, Ms. Smith buys an insurance in the amount of \$18.

Lectures 7 and 8

1. A firm's production function is $f(l, k) = l^\alpha k^{1-\alpha}$, where $0 < \alpha < 1$. It can hire labor in any quantity at wage w, but its capital stock is fixed at k_0. Each unit of capital was hired at rental rate v.

   (a) How much labor does the firm need to hire to produce q units of output?

   Solution: Since k_0 is fixed, we can find $l^*(q)$ by inverting the production function,

   $$\begin{aligned} q &= l^\alpha k_0^{1-\alpha} \\ l^\alpha &= q k_0^{\alpha-1} \\ l^*(q) &= \left(q k_0^{\alpha-1}\right)^{1/\alpha}. \end{aligned}$$

   The function $l^*(q)$ is the short-run equivalent of the conditional factor demand. Since the only way to adjust output in the short run is to adjust labor, it only depends on q.

   (b) What is the (short-run) cost of producing q units? What is the variable cost? Short-run marginal cost?

   Solution: The short-run cost is the cost of all inputs, the labor $l^*(q)$ and the capital k_0 that the firm already employs:

   $$SC(q, w, v) = w\left(q k_0^{\alpha-1}\right)^{1/\alpha} + k_0 v.$$

   The variable cost is the amount spent on labor, the factor that the firm can adjust.

$$VC(q,w) = w\left(qk_0^{\alpha-1}\right)^{1/\alpha}.$$

The short-run marginal cost is the derivative of either with respect to q.

$$SMC(q,w) = \frac{w}{\alpha} q^{\frac{1-\alpha}{\alpha}} k_0^{\frac{\alpha-1}{\alpha}}.$$

(c) What is the firm's short-run supply function?

Solution: We want the level of output $q^*(p,w)$ at which marginal cost ($SMC(q,w)$) equals marginal revenue (p),

$$\begin{aligned}
p = SMC(q,w) &= \frac{w}{\alpha} q^{\frac{1-\alpha}{\alpha}} k_0^{\frac{\alpha-1}{\alpha}} \\
\frac{\alpha p}{w} &= q^{\frac{1-\alpha}{\alpha}} k_0^{\frac{\alpha-1}{\alpha}} \\
q^{\frac{1-\alpha}{\alpha}} &= \frac{\alpha p}{w} k_0^{\frac{1-\alpha}{\alpha}} \\
q^*(p,w) &= \left(\frac{\alpha p}{w}\right)^{\frac{\alpha}{1-\alpha}} k_0.
\end{aligned}$$

(d) Why doesn't the firm's supply function depend on the cost of capital v?

Solution: The firm's supply function gives the quantity that maximizes profit. Because spending on capital is fixed at $k_0 v$, it does not vary with q and so is not related to the choice of q that maximizes profit. More technically, capital expenses are a constant term in $SC(q)$ and hence in the profit function, so they do not affect the maximization.

2. Consider a firm in the long run with production function $f(l,k) = l^{1/5}k^{3/5}$.

(a) What are conditional factor demands?

Solution: We want the amounts of labor and capital $l^c(w,v,q)$ and $k^c(w,v,q)$ that minimize cost when the firm produces q units. These are the quantities solving

$$\min_{l,k} wl + vk \ s.t. \ f(l,k) = q.$$

Setting up and solving the Lagrangian:

$$\mathcal{L} = wl + vk + \lambda(q - l^{1/5}k^{3/5})$$
$$\frac{\partial \mathcal{L}}{\partial l} = w - \frac{1}{5}\lambda l^{-4/5}k^{3/5} = 0 \frac{\partial \mathcal{L}}{\partial k} \qquad = v - \frac{3}{5}\lambda k^{-2/5}l^{1/5} = 0$$

Substituting for λ gives

$$5wl^{4/5}k^{-3/5} = \lambda = \frac{5}{3}vk^{2/5}l^{-1/5}$$
$$l = \frac{v}{3w}k$$
$$q = \left(\frac{v}{3w}k\right)^{1/5} k^{3/5}$$
$$= \left(\frac{v}{3w}\right)^{1/5} k^{4/5}$$
$$k^c(w,v,q) = \left(\frac{3w}{v}\right)^{1/4} q^{5/4}$$
$$l^c(w,v,q) = \left(\frac{v}{3w}\right)^{3/4} q^{5/4}.$$

Notice that increases in the price of each factor have an intuitive effect. When capital becomes more expensive relative to labor, k^c falls and l^c rises.

(b) What is the firm's cost function?

Solution: The firm pays w for each of the l^c units of labor it hires and v for each of the k^c units of capital it hires. The cost function is

$$C(w,v,q) = v\left(\frac{3w}{v}\right)^{1/4} q^{5/4} + w\left(\frac{v}{3w}\right)^{3/4} q^{5/4}$$
$$= q^{5/4}\left(v^{3/4}(3w)^{1/4} + w^{1/4}(\frac{v}{3})^{3/4}\right).$$

(c) What is the firm's supply function?

Solution: We want the level of quantity q that equates marginal cost with marginal revenue. Assuming perfect competition, marginal revenue is p.

Marginal cost is

$$MC(w,v,q) = \frac{5}{4}q^{1/4}\left(v^{3/4}(3w)^{1/4} + w^{1/4}(\frac{v}{3})^{3/4}\right).$$

Then

$$q^*(w,v,p) = \left[\frac{4}{5}p\left(v^{3/4}(3w)^{1/4} + w^{1/4}(\frac{v}{3})^{3/4}\right)^{-1}\right]^4.$$

(d) How would you find the firm's profit function and factor demand functions?

Solution: The factor demands are the optimal levels of labor and capital given (w,v,p). They equal conditional factor demands when the quantity of output is the optimal amount from the supply function, $q = q^*(w,v,p)$. Profit can be calculated using the amount of output from the supply function and the amounts of inputs from factor demand.

Lectures 9 and 10

1. Consider an industry where each firm has supply function $q^* = 10\sqrt{p}$ and market demand is $Q_D = 800/\sqrt{p}$. Assume that the number of firms is fixed at 5 in the short run and suppose that each firm's profit function is $10p^{3/2} - 30p$.

(a) What is industry supply?

Solution: There are 5 firms, so $Q_s(p) = 5q^*(p) = 50\sqrt{p}$.

(b) What is the short-run equilibrium (p^*, Q^*)?

Solution: The price p^* makes supply and demand in the market equal,

$$\begin{aligned} Q_S(p^*) &= Q_D(p^*) \\ 50\sqrt{p} &= 800/\sqrt{p} \\ p^* &= 800/50 = 16 \end{aligned}$$

$$Q^*(16) = 50\sqrt{16} = 200.$$

(c) What is each firm's profit in the short-run equilibrium?

> **Solution:**
> $$10(16)^{3/2} - x16 = 640 - 30(16) = 160.$$

(d) Find the price, quantity, and number of firms in the market in the long-run equilibrium.

> **Solution:** The price must make profit equal to zero,
>
> $$\begin{aligned} 0 &= 10p^{3/2} - 30p \\ p^{1/2} &= 3 \\ p^{**} &= 9 \\ Q^{**} &= 800/3. \end{aligned}$$
>
> Note that the total quantity in the market is given by demand because, with an unknown number of firms, we don't yet know market supply. To find the number of firms, divide the market quantity by the amount of output per firm,
>
> $$\begin{aligned} 10\sqrt{p} &= 30 \\ 800/3 \cdot \frac{1}{30} &= 800/90 = 80/9. \end{aligned}$$
>
> This gives the odd prediction of a non-integer number of firms. Since 80/9 falls between 8 and 9, the most likely outcome is that the ninth firm does not enter (since each firm incurs losses when there are 9 firms), so the eight firms enjoy some small profit—more than zero, but not enough to tempt another firm to enter.

Lecture 11

1. George and Karla want to trade sources (x) and documents (y). George's preferences are given by $U_G(x, y) = 3\ln(x) = \ln(y)$; Karla's are given by $U_K(x, y) = \ln(x) + 3\ln(y)$. Each enters the market with 8 units of each good.

   (a) What is the value of each endowment?

Solution: We don't know what prices are yet, but we can assume that y has price $p_y = 1$ (as a numeraire good) and x has price p. Then each endowment has value $8p_x + 8p_y = 8p + 8$.

(b) What are ordinary demands when prices are $(p, 1)$?

Solution: George and Karla solve their Lagrangians in which income is given by $8p + 8$. The results are

$$\begin{aligned} x_K^* &= \frac{8p+8}{4} = 2p + 2 \\ y_K^* &= \frac{3}{4}(8p+8) = 6p + 6 \\ x_G^* &= \frac{3}{4}(8p+8) = 6p + 6 \\ y_G^* &= \frac{8p+8}{4} = 2p + 2. \end{aligned}$$

(c) What is the price p of x in terms of y in general equilibrium? What are the final bundles?

Solution: We want the value p so that George and Karla's optimal consumptions equal the total endowments, $x_G^* + x_K^* = 16$ and $y_G^* + y_K^* = 16$. For x, this gives

$$\begin{aligned} 16 &= (2p + 2) + (6p + 6) \\ 16 &= 8p + 8 \\ 8 &= 8p \\ p &= 1. \end{aligned}$$

By Walras's law, the market for y also clears at these prices (which you may verify). The optimal bundles are $(x_G^*, y_G^*) = (12, 4)$ and $(x_K^*, y_K^*) = (4, 12)$.