Optimal Resale Policies: Third-Party Resale, Integrated Resale, or Replace with Refunds?

Preliminary and Incomplete

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Abstract

By offering refunds, ticket sellers can reallocate tickets to high-value consumers while avoiding fees paid to third-party resale platforms. So why do ticket sellers overwhelmingly allow refunds, and why have some integrated with the resale market? Using a model in which a capacity-constrained seller with rigid prices makes sales over two periods, I show that third-party and integrated resale outperform refunds when the average consumer value is uncertain. Specifically, I reach three conclusions. First, third-party resale is more profitable than refunds when there is significant aggregate uncertainty and resale fees are relatively low. When there is significant aggregate uncertainty, the seller's primary market prices may be suboptimal. It benefits from resale because resale's flexible prices adjust to reallocate tickets from low- to high-value consumers. But with little uncertainty or high fees, refunds can be better because they avoid paying fees to the resale market. Second, integrated resale offers the seller complete protection from demand uncertainty. By selling to brokers, the seller earns as much as if it could freely adjust its prices and the refund offered to early buyers. Third, consumers benefit from integrated resale and refunds when the number of consumers buying in the second period increases. The findings explain why ticket sellers have deepened ties with resellers and provide guidance on when resale is valuable and when sellers should prefer alternatives.

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1 Introduction

Resale is a significant part of the event ticket industry, with revenue expected to exceed $15 billion by 2020 (Technavio, 2017). Why do event organizers allow resale? After all, they have the ability to prohibit it altogether—for instance, by offering paperless tickets tied to a phone[1]—or replace it with refunds. Refunds are a particularly compelling option. Like resale, they protect consumers from schedule conflicts, but unlike resale, the event organizer avoids payng fees to the resale market operator. The gains from avoiding fees would be substantial: on major resale exchanges, the buyer alone pays 10–30% of the sale price in fees Office (2018).

Yet event organizers overwhelmingly choose resale over refunds, and many organizers have doubled down on resale. Professional sports teams and major universities frequently partner with resale markets StubHub (2021). Some have integrated resale into the primary market, displaying primary and resale tickets side-by-side (Ticketmaster, 2021). A few teams have even tried to limit resale to platforms they own (Zagger, 2016). Such agreements should improve resale by reducing the losses to fees, but the preference for resale remains striking.

Why do sellers allow resale instead of offering refunds, and how does integration affect the decision? In this paper, I study the choice using a two-period model of ticket sales with forward-looking consumers and stochastic preference shocks. The central finding is that resale and integrated resale outperform refunds when the average consumer value is uncertain. With uncertainty, the sellers with rigid primary market prices benefit from flexible and endogenously determined resale prices. Without uncertainty, refunds perform as well or better than both varieties of resale.

More specifically, I reach three results. First, resale on third-party platforms (which charge fees) can be more profitable than refunds when there is demand uncertainty and the seller has rigid prices. Second, integrated resale protects sellers with rigid prices from aggregate demand uncertainty, allowing them to earn as much as if they had flexible primary market prices and refunds. Third, consumers should not necessarily prefer refunds and integrated resale to third-party resale, but they are more likely to benefit when more consumers arrive in the second period.

Each result contributes to our understanding of resale and reallocation strategies. This study is the first to show that the choice between third-party resale and refunds has an ambiguous effect on profit. Earlier studies, such as Cui et al. (2014) and Cachon and Feldman (2018),

[1] The NFL is requiring paperless tickets beginning in the 2021 season (Poindexter, 2021). Some concert tours have prohibited resale by requiring attendees to present the credit card used to purchase the tickets (Pender, 2017).

found that one strategy is always superior, but neither study considered aggregate demand shocks. The study is also one of the first to consider integrated resale markets, providing a novel mechanism for why integrated resale can benefit consumers. The study of integrated markets also strengthens the idea that brokers offer protection when demand is uncertain, advanced earlier in Su (2010), by showing that brokers and resale can exactly replicate the optimal refund scheme.

The findings are valuable because they apply to a wide range of sellers and explain why different sellers choose different resale policies. The sales problem studied in this paper—that consumers purchase perishable goods in advance and then receive stochastic preference shocks—covers sellers as varied as event organizers, hotels, airlines, and fashion designers. Those sellers choose different strategies. Event organizers typically allow resale, but hotels prohibit it and offer partial refunds (implemented through cancellation fees). Crucially, the choices are consistent with the model. Hotels change prices constantly to reflect demand, making the flexibility of resale unnecessary. In contrast, sports teams face volatile demand and do not change prices to fully reflect demand swings,[2] making the flexibility of resale valuable.

The findings are also relevant to the ongoing policy debate over resale. Some states have guaranteed a right to resell tickets after some concert tours banned resale (Vozzella, 2017), and teams have faced lawsuits after limiting resale (Zagger, 2016). This paper contributes by showing that integrated resale markets and refunds do not necessarily harm consumers in a setting without aggregate demand uncertainty.

The analysis relies on a two-period model in which a monopolist seller with fixed capacity and rigid prices offers a perishable good over two periods. In the first period, strategic consumers with uncertain values choose whether to purchase in advance. The seller offers advance sales because consumers need to plan ahead to attend live events. There are two sources of demand uncertainty for consumers: (i) the possibility of schedule conflicts, which are purely idiosyncratic, and (ii) news about quality, like injuries to a team's star player, that affect all consumers. In the second period, consumers learn their final values. Consumers who purchased in advance decide whether to resell or request a refund; additional consumers arrive and decide whether to purchase. With refunds, some consumers return the good and the seller puts the recovered units back on sale in the primary market. With resale, consumers make their tickets available in a resale market where prices are determined by market-clearing. Both strategies are profitable when the seller has limited capacity because they transfer tickets from low-value consumers to

[2]The assumption that prices are rigid in the model captures the fact that, even if sellers use dynamic pricing, they do not change prices enough to account for demand shocks. I expand on the argument in Section 2.

high-value consumers without tickets.

When choosing between third-party resale and refunds, the key tradeoff is between price flexibility and resale fees. Primary market prices are often inflexible, for example because prices are printed on the tickets. The inflexibility is harmful for refunds because all sales are made at primary market prices. Consider a sports team that offers a refund and learns that its star player is injured. Consumers have lower values after the injury, so some consumers will request refunds but the seller will be unable to sell all the recovered tickets. In contrast, resale prices are flexible and fall after the injury. The seller profits from the additional resale transactions by charging consumers for their expected resale revenue, but surrenders some of the gains to the resale market operator as fees. I show that the optimal strategy depends on the degree of uncertainty and size of the fees. The emphasis on price flexibility is reasonable because consumer values change dramatically based on the performance of sports teams or quality of theater productions, which may be unknown at the time of purchase. For example, Figure 1 shows that annual resale prices for five NFL teams vary by over \$100—and often \$200—for each team.

Integrated resale allows the seller to set the resale fee and avoid paying it to a third party. It is unsurprising that doing so is profitable. More strikingly, if the seller uses brokers, it can earn as much as if it could frictionlessly adjust its prices and the amount refunded.

In a simplified setting without aggregate demand shocks,[3] the welfare effects of each strategy revolve around the optimal fee charged by the seller and a third-party resale platform. With third-party resale, the resale platform faces a classic monopoly problem, trading off the volume of resale against the amount earned on each transaction. In contrast, with integrated resale or refunds, the seller only uses the fee to deter strategic consumers from waiting to purchase until the second period. As the fee rises, the supply of tickets in the second period falls, allowing the seller to extract more surplus from early arrivals. When there are many late arrivals, the seller's incentive to raise the fee is weaker, allowing refunds and integrated resale to be optimal for consumers and society.

The paper proceeds as follows. First, I review the related literature. I outline the model in Section 2 before presenting additional assumptions and preliminary results in Section 3. I compare the performance of third-party resale and refunds in Section 4, then extend it in Section 5 to consider integrated resale. I study the effect of each strategy on welfare without aggregate demand uncertainty in Section 6. I conclude in Section 7.

[3] In this setting, allocations and welfare with integrated resale and refunds are identical.

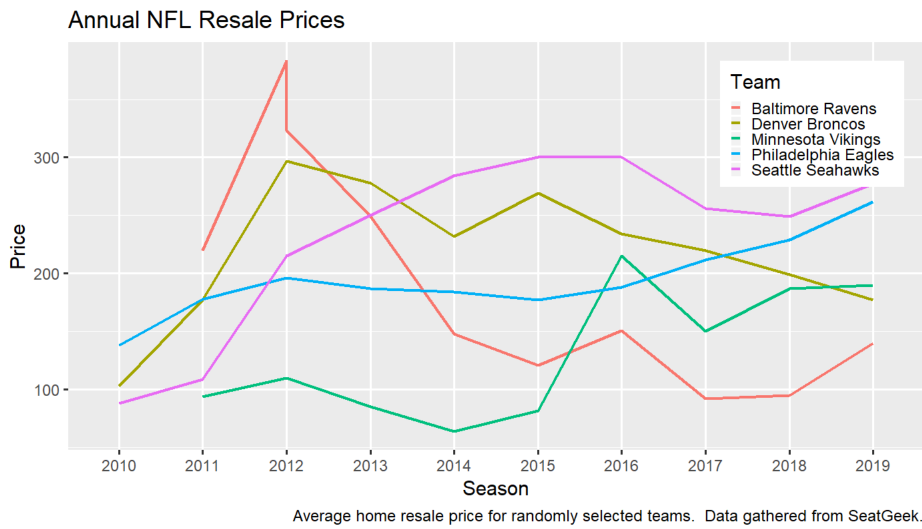


Figure 1: Annual NFL resale prices show substantial variation.

Related Literature. There is a significant body of work investigating whether resale is more profitable than offering no method of reallocation. Courty (2003) studies a model with no capacity constraints and shows that resale cannot strictly increase profit. Karp and Perloff (2005) find that resale raises profit in a setting where scalpers can price discriminate but the seller cannot. Cui et al. (2014) show that a capacity-constrained seller can benefit from both resale and scalpers, but emphasize that refunds (equivalently, options) would be more profitable. Zou and Jiang (2020) focus on the effects of integrated resale markets and find that they can increase profit and consumer welfare. Su (2010) shows that scalpers can increase profit when there is aggregate demand uncertainty.

Several studies have also considered resale empirically. Leslie and Sorensen (2014) find that resale promotes efficiency, but that gains are partially offset when scalpers try to purchase underpriced tickets. Lewis et al. (2019) measure the value of resale for season ticket buyers in professional baseball. Waisman (2021) studies the choice of selling mechanism, like auctions and posted prices, in online resale markets for event tickets.

This paper differs from earlier work on ticket resale in three respects. First, this paper considers why a seller should prefer resale to other methods of reallocation, like refunds. Other strategies produce similar benefits, yet there have been few comparisons, with Cui et al. (2014) as the most notable exception. However, Cui et al. (2014) find that resale is strictly worse than refunds, which is puzzling given the continued use of resale. This paper contributes by showing that resale can be more profitable when sellers have rigid prices and there is aggregate demand uncertainty. Cachon and Feldman (2018) also compare resale with refunds, finding that resale is always better in an environment with only two potential buyers. Vollmer (2021) conducts an empirical analysis of third-party resale and refunds based on the theory in this paper, concluding that refunds are more profitable for a seller of college football tickets.

Second, this paper emphasizes the effects of integrated resale markets, a focus shared with Zou and Jiang (2020). I show that the seller achieves its optimal profit by integrating and charging a fee; it does strictly worse under the common assumptions of no fees and third-party resale. Although Zou and Jiang (2020) also consider integration, the forces driving their results are entirely different. In their model, the seller benefits from integrated resale by intentionally creating a shortage in the first period; the seller does not benefit at all from third-party resale. By contrast, in this paper, there are no shortages and both integrated and third-party resale are profitable because of the chance to reallocate. The differences lead to distinct explanations for the effects of integration on profit and consumer welfare. Consumers benefit in Zou and Jiang (2020) because the seller increases event capacity despite charging high fees; in this paper, event capacity is fixed and consumers benefit because the seller chooses lower fees when there are more late arrivals.

Third, this paper emphasizes the importance of aggregate demand shocks. It is common for studies to have idiosyncratic shocks that cause consumers with tickets to want to resell, but aggregate shocks are necessary to demonstrate the value of resale's flexible prices. The benefits of flexible prices are also emphasized in Su (2010), which shows that scalpers can increase profit and, in a special case, that scalpers and resale are as profitable as flexible prices. This paper extends the result by showing that integrated resale and brokers are better than flexible prices—they allow the seller to replicate the optimal refund scheme.

Another branch of the literature has considered the benefits of refunds. Gallego and Şahin (2010) establish that refunds can be valuable in the context of revenue management. Xie and Gerstner (2007) show that refunds can be profitable in a simpler setting.

Refunds can also be analyzed as consumer options, as in Sainam et al. (2010) and Alexandrov and Bedre-Defolie (2014). Consumer options focus on a setting with heterogeneous preferences over uncertain future states, like fans of different teams who only want to see their team in the next round of a tournament. I focus on a separate setting in which there is aggregate uncertainty over a shared component of values.

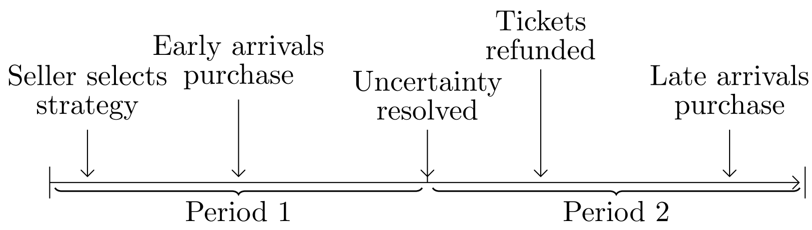


Figure 2: Model timeline.

2 Model

A monopolist seller with fixed capacity K and marginal costs normalized to zero sells a product over two periods. A measure $a_1 < K$ of buyers arrives in the first period and an additional measure a_2 arrives in the second, where $a_1 + a_2 > K$. Consumers are strategic: those who arrive in the first period can wait to purchase until the second period at no cost. To match the event ticket setting, I assume that the seller makes sales in the first period.[4]

Consumer i's final value for the product is $V + s_i$, where S and V are random variables whose realizations are learned at the start of the second period. The consumer-specific component of values s_i is independently drawn from distribution $G(s)$ and has $\mathrm{E}(S) = 0$. I assume that $G(s)$ has a density $g(s)$ satisfying $g(s) > 0$ at all points in its support. There is a single realization of the random variable V for all consumers. It takes value V_L with probability $\Pr(V_L) = \alpha$ and value $V_H > V_L$ with probability $\Pr(V_H) = 1 - \alpha$. When $\alpha \in \{0, 1\}$, aggregate demand is said to be certain.

The structure of consumer values captures that there is both idiosyncratic and aggregate uncertainty. Idiosyncratic shocks, captured in the random variable S, are common in studies of resale. They capture that some consumers want to attend but end up unable to for reasons that do not affect other consumers, like illness or work conflicts. Aggregate shocks, captured in V, capture that average values for the event may change over time. For example, values will fall if a star player gets injured, or they may rise if a new musical earns sparkling reviews.

Under each strategy, the seller sets a menu of prices $\{p_1, p_2\}$ at the start of the first period to maximize profit. (When offering refunds or using integrated resale, it has additional choice variables.) The price p_1 is charged in the first period and p_2 in the second. Like many event organizers, the seller is unable to revise prices after they have been set. The early price p_1 can be interpreted as a season ticket price for sports or theater (since ticket packages are sold well

[4] The assumption keeps the analysis focused on the resale and refund strategies, but the seller is highly likely to prefer advance sales anyway. The seller always prefers integrated resale to making no advance sales and, when demand is certain, always prefers refunds.

in advance) or a special advance price for concerts. Under this interpretation, the price p_2 is the face value of the ticket, which is charged afterwards.

The assumption of complete price rigidity is appropriate for the event ticket industry. Ticket sellers may want to revise prices but lack the technology and data to do so. For example, the university studied in Vollmer (2021) expressed a desire in dynamic pricing but continued to use physical tickets, stamped with and sold at a price selected before the season. For perishable goods more generally, sellers have a short time in which to change prices, and they may be unable to raise the price of popular goods before they sell out.

The most sophisticated sellers, like professional sports teams, now use dynamic pricing.[5] However, price rigidities still approximately capture their sales problem because they do not fully adjust their prices to reflect demand. Consider that NFL teams smoothly increase primary market prices even though the resale prices from Figure 1 are extremely volatile.[6] The habit of smoothly adjusting prices creates mispricing in the primary market; the assumption that prices are rigid captures the mispricing.

The seller chooses among three strategies: offering refunds, allowing third-party resale, and allowing integrated resale.

Refunds. When offering a refund contract, the seller chooses a refund r in addition to its menu of prices (p_1, p_2). Consumers can return the good for a payment of r at the start of the second period. (Like prices, the refund cannot be changed in the second period.) The seller puts the recovered units back on sale at price p_2. The seller's optimal menu with refund contracts is $\{p_1^{RC}, p_2^{RC}, r^{RC}\}$.

Third-Party Resale. With third-party resale, consumers have the right to resell to each other in secondary markets in the second period. The resale price $p_2^r(p_2, V)$ clears the resale market in the second period given purchases in the first period, prices, and the realization of V. The resale market is operated by a third party that collects a fee τ on each transaction: buyers pay $p_2^r(p_2, V)$, but resellers only receive $p_2^r(p_2, V) - \tau$. The fee is modeled as exogenous from the seller's perspective because it is set by the resale market operator.[7] The seller's optimal prices with secondary markets run by a third-party are $\{p_1^{SM}, p_2^{SM}\}$.

Integrated Resale. When the primary and resale markets are integrated, consumers still have the right to resell at a market-clearing price $p_2^r(p_2, V)$. The difference from third-party resale

[5] I use dynamic pricing to refer to the ability to change prices at any time. In the model, the seller can set a different price in each period but cannot change prices after learning about shocks.

[6] See Appendix A for NFL primary market price paths.

[7] It is endogenous with integrated resale and, in Section 6, I consider the optimal third-party fee.

is that the primary market seller sets the fee τ and collects it on each transaction. The seller's optimal menu with integration is $\{p_1^I, p_2^I, \tau^I\}$.

3 Preliminaries and the Seller's Problem

Before presenting the main results, I introduce additional assumptions that clarify the seller's problem and explain why the seller benefits from reallocation. I start with efficient rationing.

Assumption 1 Efficient Rationing. Whenever tickets are rationed, they are allocated to the consumers with the highest values $V + s_i$ who do not already have the good.

Assumption 1 simplifies the analysis by making it unprofitable for the seller to create shortages. Although shortages are compatible with profitable resale (e.g. Zou and Jiang (2020)), the core benefit of resale in this paper is to reallocate goods from low- to high-value consumers. Efficient rationing keeps the focus on reallocation and is commonly used, as in Cui et al. (2014) and Su (2010). The assumption slightly enhances the performance of refunds,[8] a tolerable consequence since this paper focuses on the benefits of resale.

Efficient rationing simplifies the seller's profit functions. Start with refund contracts. The seller chooses a menu $\{p_1, p_2, r\}$ where consumers who bought a ticket at $t = 1$ can return it for r at $t = 2$. Consumers who arrive in period one have the same expected value before shocks arrive, which considers the value of using the ticket, the value of the refund, and the ability to wait and purchase in the second period. The seller charges the highest price possible for advance sales, expected surplus from purchasing in advance minus expected surplus from waiting,

$$\begin{aligned} p_1^{RC}(p_2, r) = \sum_{v \in \{V_L, V_H\}} \Pr(v) \Big(\int_{r-v}^{\infty} v + s\, dG(s) + \int_{-\infty}^{r-v} r\, dG(s) - \\ \int_{s^{*RC}(p_2, r, v)}^{\infty} v + s - p_2\, dG(s) \Big). \end{aligned} \tag{1}$$

The sum considers the two possible realizations of V. The first term in parentheses captures surplus from using the ticket when the consumer's draw of S is high; the second captures surplus from requesting a refund when S is low. The last term is the surplus that could be earned by waiting to purchase in the second period. The value $s^{*RC}(p_2, r, V)$ is the lowest type rationed a ticket under Assumption 1 when demand outstrips supply, like when $V = V_H$ and the seller's

[8]The seller earns more revenue from shortages with efficient rationing, and shortages are more likely with refunds because of price inflexibility.

price p_2 is low. When there is no rationing, anyone willing to pay p_2 acquires a ticket and $s^{*RC}(p_2, r, V) = p_2 - V$.

At price $p_1^{RC}(p_2, r)$, all a_1 early arrivals buy in advance. The seller issues refunds at cost r for early buyers with $V + S < r$, amounting to $a_1 G(r - V)$ units. It cannot sell more than $(K - a_1) + a_1 G(r - V)$ in the second period, giving profit

$$\begin{aligned} \pi^{RC}(p_2, r) = \; & a_1 p_1^{RC} + \sum_{v \in \{V_L, V_H\}} \Pr(v) \Big(- a_1 G(r - v) r + \min\{a_2(1 - G(p_2 - v)), \\ & K - a_1(1 - G(r - v))\} p_2 \Big). \end{aligned} \tag{2}$$

Equations (1) and (2) highlight the forces affecting the seller: a higher price p_2 may reduce the quantity sold in the second period, but it raises the price in the first period by making waiting less attractive. A higher refund r can increase profit by reducing misallocation to early arrivals who have low realizations of S, but it lowers the optimal price in the second period so the seller can clear the recovered inventory. Uncertainty affects the seller by changing the number of consumers who request a refund or want to purchase at price p_2 in the second period. When the realizations V_L and V_H are far apart, the quantities refunded and sold in the second period could differ significantly.

Next consider third-party resale. The seller chooses prices $\{p_1, p_2\}$, where the price p_1 is still chosen to extract all surplus from early arrivals. Early arrivals earn surplus from using the ticket or reselling; they can wait to purchase resold tickets or primary market tickets. Let $p_2^r(p_2, V)$ denote the resale price when the seller sets price p_2 and the realized value is V. The seller's optimal price $p_1^{SM}(p_2)$ is

$$\begin{aligned} p_1^{SM}(p_2) = \sum_{v \in \{V_L, V_H\}} \Big(& \int_{p_2^r(p_2,v) - \tau - v}^{\infty} v + s \, dG(s) + \int_{-\infty}^{p_2^r(p_2,v) - \tau - v} p_2^r(p_2, v) - \tau \, dG(s) - \\ & \int_{s^{*SM}(p_2,v)}^{\infty} v + s - p_2 \, dG(s) - \\ & \int_{p_2^r(p_2,v) - v}^{s^{*SM}(p_2,v)} v + s - p_2^r(p_2, v) \, dG(s) \Big) \Pr(v). \end{aligned} \tag{3}$$

The value of resale depends on V through the resale price. Similarly, wait surplus depends on the realization of V. When the realized value is V_H, it is possible that the market-clearing resale price exceeds the seller's price, $p_2^r(p_2, V_H) > p_2$, in which case the $K - a_1$ remaining primary market tickets are underpriced and rationed to consumers with values $S \geq s^{*SM}(p_2, V_H)$.[9] When there is no rationing, $p_2^r(p_2, V_H) \leq p_2$, then s^{*SM} is infinitely large. Profit is

$$\begin{aligned}\pi^{SM}(p_2) = \; & a_1 p_1^{SM}(p_2) + \sum_{v \in \{V_L, V_H\}} \min\{\big(a_2(1 - G(p_2 - v)) - a_1 G(p_2^r(p_2, v) - \tau)\big)^+, \\ & K - a_1\} \Pr(v) p_2.\end{aligned} \tag{4}$$

There are three key differences from the problem with refunds. First, third-party resale incurs a fee τ on each unit paid to the resale market operator. Second, some transactions take place at the endogenously determined resale price $p_2^r(p_2, V)$ rather than the seller's price p_2. Since the seller charges consumers their expected resale revenue in the first period, it earns the resale price minus fees on each unit reallocated. Third, the resale price and fee determine the extent of reallocation, which varies across realizations of V. Refunds, in contrast, give the seller direct control but do not change with demand. With both strategies, the seller faces a tradeoff between the price charged to early arrivals and the number of units sold later.

Profit in equation (4) depends on the resale price $p_2^r(p_2, V)$, which clears the resale market. When the number of willing resellers exceeds the number of buyers at the seller's price p_2, $a_1 G(p_2 - \tau - V) > a_2(1 - G(p_2 - V))$, the resale price must fall, leading to $p_2^r(p_2, V) < p_2$ and no primary market sales at $t = 2$. When the number of resellers plus primary market inventory is less than demand at p_2, $K - a_1 + a_1 G(p_2 - \tau - V) < a_2(1 - G(p_2 - V))$, the resale price must rise, leading to $p_2^r(p_2, V) > p_2$ and rationing of all $K - a_1$ units in the primary market. In all other cases, the resale and primary market prices are equal. The $a_1 G(p_2 - \tau - V)$ units in the resale market are sold first in equilibrium (if not, the resellers would want to undercut the price) and the remaining units are sold in the primary market.

Integrated resale is similar to third-party resale, but differs in two respects. First, the seller chooses the fee τ charged for resale. And second, the fee is paid to the seller because it operates the resale platform. The price in the first period p_1^I is the same as with third-party resale except that it is now a function of τ. Profit is

$$\begin{aligned}\pi^{I}(p_2, \tau) = \; & a_1 p_1^{SM}(p_2, \tau) + \sum_{v \in \{V_L, V_H\}} \min\{\big(a_2(1 - G(p_2 - v)) - a_1 G(p_2^r(p_2, v) - \tau)\big)^+, \\ & K - a_1\} \Pr(v) p_2 + a_1 G(p_2^r(p_2, v, \tau) - \tau)\tau.\end{aligned} \tag{5}$$

[9] The cutoff $s^{*SM}(p_2, V_H)$ solves $K - a_1 = a_2(1 - G(s^{*SM}(p_2, V_H)))$.

The next set of assumptions explains why the seller benefits from reallocation. I make a standard hazard rate assumption and additional regularity assumptions on the distribution of the idiosyncratic component of values S.

Assumption 2. $G(s)$ has a weakly decreasing hazard rate $\frac{1-G(s)}{g(s)}$ and a differentiable positive log-concave density $g(s)$.

Assumption 2 provides regularity conditions for the seller's choice of prices[10] and guarantees that the distribution $G(s)$ is invertible. The hazard rate assumption is standard for pricing. Common distributions like the exponential, uniform, and normal meet all of the requirements.

The next assumption provides a condition outlining when the seller benefits from reallocation. The seller benefits when it can transfer the ticket from a low-value consumer to a high-value one that would have otherwise gone unserved. The gains increase in the arrival rates a_1 and a_2. An increase in a_1 means fewer tickets remain in period two and hence more unserved consumers if there is no reallocation. An increase in a_2, in turn, raises the value of the marginal unserved consumer. The formal condition is given in Assumption 3 and its effect is proven in Lemma 1.

Assumption 3. Arrivals a_1 and a_2 satisfy

$$\begin{aligned} K(\alpha V_L + (1-\alpha)V_H) + (K-a_1)G^{-1}(\beta) < \max\{&\alpha a_1(V_L + G^{-1}(\gamma)) + (1-\alpha)K(V_H + G^{-1}(\gamma)), \\ &\alpha K(V_L + G^{-1}(\gamma)) + (1-\alpha)\big(a_1\beta(V_H + G^{-1}(\gamma)) + (K - a_1\beta)(V_L + G^{-1}(\gamma))\big)\}, \end{aligned} \tag{6}$$

where $\beta = \frac{a_1+a_2-K}{a_2}$ and $\gamma = \frac{a_1+a_2-K}{a_1+a_2}$.

Assumption 3 holds when a_1 and a_2 are sufficiently large. As a_1 approaches K, the right-hand side approaches $K(\alpha V_L + (1-\alpha)V_H + G^{-1}(\gamma))$ and the inequality will hold as long as $G^{-1}(\beta)$ is close to $G^{-1}(\gamma)$. The two can be made arbitrarily close by choice of a_2.

Lemma 1. *The seller prefers resale to no reallocation when $\tau = 0$.*

Assumption 3 is consistent with observed event ticket markets, where a large share of event tickets are sold early (analogous to a large value of a_1) and sellers permit reallocation. Season tickets, often sold months before the season begins, represent 70–80% of total capacity for the average NFL and NBA team (Hubbard, 2017). Leslie and Sorensen (2014) find that an average of 70% of tickets for major concerts are sold within a week of going on sale.

[10] Appendix Lemma 1 establishes that the first order condition is monotone in the price p_2 when there is no demand uncertainty

Although the result does not directly address refunds, it demonstrates that the seller always prefers to reallocate in some form when fees are low. Moreover, the seller may not prefer resale to refunds—determining which is best is a focus of the remainder of the paper. The seller never prefers not reallocating to a refund because it can offer a refund of zero.

The final assumption, Assumption 4, implies that the seller benefits from selling all units when demand is certain.[11] The assumption is consistent with the basis for reallocation, that transferring units can be profitable when there are no additional units to sell. It is also helpful analytically, implying that the seller chooses the highest price allocating all units when demand is predictable.

Assumption 4.

$$V_L \geq \underline{V} \equiv \frac{(a_1 + a_2)(1 - G(s_H(\tau))) - a_1 G(s_H(\tau) - \tau)}{a_2 g(s_H(\tau)) + a_1 g(s_H(\tau) - \tau)} - s_H(\tau), \tag{7}$$

where the value $s_H(\tau)$ solves

$$K - a_1 + a_1 G(s_H(\tau) - \tau) = a_2(1 - G(s_H(\tau))). \tag{8}$$

4 Third-Party Resale and Refunds

The discussion in Section 3 identified three factors that determine the relative profitability of resale and refunds: resale incurs fees that are absent with refunds, refunds give the seller control over the extent of reallocation, and resale prices adjust to changes in V while primary market prices do not.

The effect of resale fees is straightforward: they reduce the revenue the seller gains from each unit resold. The effects of the other factors depend on the degree of demand uncertainty. The ability to choose the level of the refund should be an advantage over resale, where the amount of reallocation depends on fees and the resale price. Choosing the level of the refund lets the seller balance the gains of reallocation against a desire to keep prices high. However, the refund may not be effective if demand is uncertain. For example, if the seller offers a modest refund and the value turns out to be high, then few consumers—even those with relatively low values—may return their tickets, leaving high-value consumers unserved. And if the seller's refund is high relative when demand is low, even consumers with high values will return their tickets.

In contrast, resale's price flexibility is an asset for the seller when demand is uncertain. Even

[11]For a formal proof, see Appendlix Lemma 2.

when realized demand is unexpectedly high or low, resale allows reallocation because consumers base decisions to buy or resell on the resale price. The seller's price p_2 may still be suboptimal with resale, but its impact is dampened by the availability of resale tickets.

When there is no demand uncertainty, however, it is not surprising that resale offers few advantages. In fact, refunds are strictly more profitable.

Lemma 2. *When demand is certain, profit is strictly higher with refunds than with resale.*

Lemma 2 demonstrates that refunds are strictly better than resale for certain demand because refunds avoid fees and allow the seller to control the extent of reallocation. The seller optimally chooses a refund lower than its price in period two so that it can increase the price for early arrivals. The consequence is that some early arrivals will use their tickets but have lower values than unserved consumers.[12]

The logic of Lemma 2 extends to the case where demand uncertainty is mild. If the two realizations are close enough together, or if one realization is sufficiently unlikely, then refunds will continue to be more profitable. I formalize the result in Proposition 1.

Proposition 1. *Refunds are more profitable than resale if*

1. $V_H - V_L \leq \underline{\Delta}$ *for some* $\underline{\Delta} > 0$, *or*
2. $\alpha \leq \underline{\alpha}$ *for some* $\underline{\alpha} > 0$, *or*
3. $\alpha \geq \bar{\alpha}$ *for some* $\bar{\alpha} < 1$.

In the opposite case where demand uncertainty increases, either because the values V_L and V_H are far apart or because both outcomes are equally likely, the advantages of resale emerge. If the seller offers a refund and V_L and V_H are far apart, the seller's price and refund must either be too low for the high demand state, leaving money on the table, or too high for the low demand state, resulting in many refunds but few sales. Resale eases the constraint by allowing some transfers at the resale price. When demand uncertainty is sufficiently large, the seller prefers resale.

Proposition 2. *Let* $\alpha \in (0,1)$ *and define*

$$h(v) = \alpha(V + s_L(\tau)) + (1-\alpha)(K(V + s_H(\tau)) - \pi^{RC})$$
$$\underline{V}_L = \inf\{V : h(v) > 0, \alpha(a_1 \int_{-V_L}^{\infty} V_L + s\, dG(s) - a_1 V_L < h(v)\},$$

[12] The resulting misallocation is the main distortion due to monopoly power in the model. There is no quantity distortion because the seller wants to sell all of its inventory.

where $s_L(\tau)$ *solves* $a_1 G(s_L(\tau) - \tau) = a_2(1 - G(s_L(\tau)))$.

For any $V_L > \max\{\underline{V}, \underline{V}_L\}$, *there exists* $\bar{V}_H(V_L)$ *such that resale is more profitable than refunds when* $V_H \geq \bar{V}_H(V_L)$ *and* $\tau = 0$. *For each* $V_L > \underline{V}_L$ *and* $V_H \geq \bar{V}_H(V_L)$, *there exists* $\bar{\tau}(V_L, V_H) > 0$ *such that resale is more profitable when* $\tau < \bar{\tau}$.

Proposition 2 shows that when there is any uncertainty over the demand state, there are values V_L and V_H such that resale is more profitable when fees are sufficiently low.

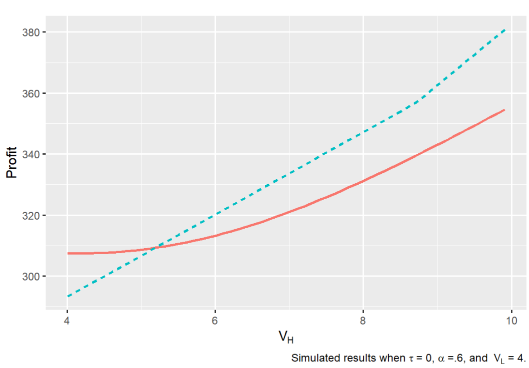
Decisions to use resale and refunds are largely in line with the results. Sellers with rigid prices that face substantial demand uncertainty, like the NFL teams in Figure 1 or other sellers of event tickets, tend to use resale. But sellers with flexible prices or demand that can be predicted in advance, like hotels, opt for refunds.

The required assumptions are relatively mild: resale can be more profitable for any probability α, but the low value V_L must be high enough that the seller suffers by setting a high price p_2 and making few sales when demand is low.

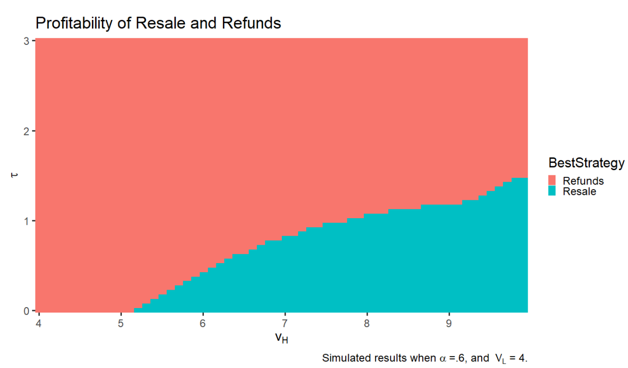
Example. Suppose $K = 50$, $a_1 = 37$, $a_2 = 150$, $V_L = 4$, $\alpha = .6$, and $S \sim N(0, 9)$. I simulate the market for $4 \leq V_H \leq 10$ and $\tau \leq 3$, with results presented in Figure 3.

The pattern in Figure 3a for profit is in line with Propositions 1 and 2: refunds are initially more profitable when aggregate uncertainty is low, but resale becomes more profitable as V_H grows and uncertainty increases. Tellingly, it does not take much uncertainty for the seller to prefer frictionless resale—resale becomes more profitable when V_H is just over 5.

Even as fees rise, resale can be more profitable. Figure 3b shows which strategy is more profitable for each V_H-τ combination. As V_H moves away from V_L, resale becomes more profitable at higher levels of the fee.



(a) Profit when $\tau = 0$



(b) Best strategy as V_H and τ vary

Figure 3: Results for the simulated example.

5 Integrated Resale

The results of Section 4 explain the use of resale but do not fully account for the steps taken by sellers of event tickets. Sellers are not only choosing resale, but also starting their own resale platforms. In this section, I investigate the gains to doing so and show that sellers can earn as if they offered a flexible menu of refunds.

In this section, suppose the seller owns and operates the resale market. As the resale market operator, the seller chooses the level of the resale fee and earns any fees paid. The difference from resale profit in equation (4) is that the mass $a_1 G(p_2^r(p_2, V) - \tau - V)$ of consumers who resell now contribute $p_2^r(p_2, V)$ in revenue because the seller keeps the fees.

Although fees are less important to profit when they are not paid to a third-party operator, they still affect profit by determining which consumers resell. Ownership of the resale market thus gives the seller control over the extent of reallocation, like the ability to select the level of the refund. In fact, control of the extent of reallocation is greater when choosing the fee than when choosing a refund. The fee defines reallocation relative to the resale price, causing consumers with values below $p_2^r(p_2, V) - \tau$ to give up the good. The amount of reallocation is similar at each realization of V because the resale price adjusts. Refunds, on the contrary, are at a fixed level that may be suboptimal for realized demand. For instance, a refund that is optimal for the low value V_L may result in fewer than optimal returns when the realization is V_H.

In all, owning the resale market removes the chief drawback of resale, fees paid to a third-party operator, while gaining a benefit of refunds, the ability to choose the extent of reallocation. The sum of the changes is that sellers who control the resale market prefer resale to refunds.

Proposition 3. *The seller earns weakly more when it owns and operates a resale market than when it offers a refund.*

Proposition 3 demonstrates that resale not only can be better than refunds, but that it is better when the seller controls the resale market. The conclusion helps explain why event organizers have embraced resale instead of banning it. The Minnesota Timberwolves NBA team, for example, has attempted to limit resale to a platform where it collects fees (Zagger, 2016). The Dallas Cowboys NFL team has pursued a slightly different path, partnering with the resale market SeatGeek in exchange for an equity stake in the company (Rovell, 2018).[13] Owning the resale market is superior to partnerships in the model, but partnerships may realize most

[13] Sports leagues may also receive payments in exchange for deals to run the official resale market, as in the NFL and NBA's deals with Ticketmaster, but terms are not disclosed.

benefits without the fixed costs of developing a resale platform.

The increase in profit in Proposition 3 is strict under most circumstances. It is only possible to earn the same amount with refunds if (i) the seller finds it optimal to set its price and fee as if V_H were certain and (ii) the resale price in the low state and high state are equal. Both conditions require limited demand uncertainty and the first condition requires the low state to be relatively improbable.

But Proposition 3 undersells the potential gains from using resale. The seller's core problem is that it faces price rigidities or menu costs. When the seller owns the resale market and sells to brokers—agents who purchase for the sole purpose of reselling—the seller earns as if it could costlessly adjust prices.

Assumption 5. There is an unlimited number of atomistic brokers in the market. The seller offers them tickets at a price specifically designated for brokers.

With brokers, the seller is able to sell a_1 tickets to early arrivals and its remaining $K - a_1$ tickets to brokers, moving all transactions to the resale market in the second period. Because brokers are atomistic, the seller can set a price extracting all of their expected resale revenue, the resale price minus the fee. The seller contracts directly with brokers at that price and brokers accept because expected resale revenue is less than the price charged to early arrivals.[14]

Proposition 4. *When the seller owns the resale market and sells to brokers in the first period, it earns as much as if it offered a menu of flexible prices and refunds,* $\{p_2^{RC}(V), r^{RC}(V)\}$.

Corollary 1. *Profit under resale with ownership and sales to brokers is strictly higher than with refunds when demand is uncertain,* $\alpha \in (0, 1)$.

Strikingly, Proposition 4 shows that sellers with perfectly adjustable prices are willing to allow resale if they operate the resale market and can sell to brokers. Corollary 1 clarifies that doing so is strictly better than a typical refund contract when there is any demand uncertainty.

The prediction that sellers should contract with brokers and embrace resale markets resonates because many sellers have done so. In addition to the resale partnerships and ventures mentioned earlier, sports teams sell to brokers. The University of Kansas has a contract with a broker for men's basketball tickets (Shepherd, 2019). Professional sports teams often sell season ticket packages to brokers, and brokers are the majority of season ticket buyers for some teams (Hubbard, 2017).

[14] In equation (3), s^{*SM} is infinitely high when all units are sold in the first period and all sales in the second occur at the resale price. The price exceeds expected resale revenue $\mathrm{E}(p_2^r(p_2, V) - \tau)$.

6 Welfare

While revenue is the primary consideration, ticket sellers should also value consumer welfare. Sellers have ongoing relationships with consumers who purchase ticket packages each year. Sellers also benefit from the emotional attachment consumers have to a team. Policies that harm consumers may imperil the attachment and reduce demand in the long term.

Concerns over consumer reactions are not hypothetical. Sports teams that put limits on resale have faced class action lawsuits (Zagger, 2016). Concert tours that prohibited resale have even inspired laws guaranteeing a right to resell (Vozzella, 2017). It would be unwise to change resale policies without considering the effects on consumers. In this section, I consider the effects of resale and refunds on consumer and social welfare.

The welfare effects are ambiguous, but third-party resale market operators are worse for consumers than refunds and seller-owned resale when the number of arrivals in period two a_2 is large. To make the analysis tractable, I focus on the case with no demand uncertainty, $V_L = V_H$.[15] Demand uncertainty makes analysis difficult because there is no closed form for optimal prices, and welfare effects remain ambiguous with uncertainty.[16]

The main insight for welfare analysis is that welfare only depends on the gap between the highest-value consumer who does not attend the event (an arrival in period two) and the lowest-value consumer who does attend (an arrival in period one). The gap, which I refer to as the distortion, emerges because consumers who would not purchase in the second period may not receive enough compensation to return the ticket for a refund or resell.

Definition 1. The *distortion* δ is $\delta \equiv \tau$ for resale and $\delta \equiv p_2^{*RC} - r^{*RC}$ for refunds.

Lemma 3. *Total and consumer welfare only depend on the distortion δ. Both decrease in δ.*

The relationship between the distortion and total welfare is intuitive. Total welfare is maximized when all tickets are allocated to consumers with the highest values, which happens when there is no distortion. Consumer welfare depends on the distortion indirectly through the price in the second period. Surplus for arrivals in the second period depends on the price p_2; surplus for arrivals in the first also depends on p_2 in equilibrium because that is the surplus they could earn by waiting to purchase. Since smaller distortions increase supply in the second period, they lower the price and raise consumer welfare.

[15]Without uncertainty, the seller sets the highest price that exhausts inventory. The resale price equals the price charged in the primary market in equilibrium.

[16]Resale would be expected to be better with uncertainty because of price flexibility, but the seller could still choose a lower price with refunds that grants extra surplus to consumers in the high state V_H.

The discussion of Lemma 2 shows that the seller wants to introduce a distortion with refunds, $\delta^{RC} > 0$, and therefore also with ownership of the resale market. If τ is an exogenous parameter, the welfare-maximizing strategy reduces to whether $\tau < \delta^{RC}$.

Although the assumption that τ is exogenous is appropriate for major resale markets like StubHub, which have policies that apply to a wide range of events, it is not informative about the resale market operator's likely level of fees. To learn about welfare at plausible values of τ, I suppose that the resale market operator sets its fee to maximize profit from the event studied. Assuming that the resale market operator has no marginal costs, its profit is

$$\pi^{\mathrm{RMO}}(\tau) = \tau a_1 G(p_2^{SM}(\tau, v) - \tau - v), \tag{9}$$

where $p_2^{SM}(\tau, v)$ is the seller's optimal price in the second period, now dependent on τ.

The resale market operator's problem involves the classic pricing tradeoff between quantity sold and price per unit. The operator has an obvious incentive to introduce a distortion—it would earn no revenue without one. In contrast, a seller that offers refunds or owns the resale market only introduces a distortion to limit supply in the second period and charge a higher price in both periods.

The incentive to distort with refunds and resale market ownership responds strongly to the elasticity of the price in period two. When the elasticity is low, the price in period two hardly moves in response to a higher refund, reducing the return to low refunds. The resale market operator, in contrast, has little incentive to lower its fee based on the elasticity. As the number of arrivals in the second period a_2 increases, the price elasticity falls and welfare with refunds and ownership passes welfare with a resale market operator.

Proposition 5. *Suppose that S has compact support $[\underline{s}, \bar{s}]$ and $g(s)$ is continuous. There exists $\bar{a}_2$ such that total and consumer welfare are higher with refunds when $a_2 > \bar{a}_2$.*

The lesson of Proposition 5 is that changing policies, either to refunds or to a new resale market, may benefit consumers. Sellers who wish to change policies can ease the transition if the new fee is lower than what would be charged by StubHub or its competitors.

Example. Figure 4 shows the optimal distortion with resale and refunds when S is uniformly distributed on $[-.5, .5]$, $V = 1$, $K = 1$, and $a_1 = .95$. At low levels of a_2, the distortion with refunds (and ownership) is significantly higher than with resale. The refund distortion falls rapidly in a_2, ending over 50% lower than its initial level, and markedly lower than with a third-party resale market operator. The resale market operator's optimal fee does not vary

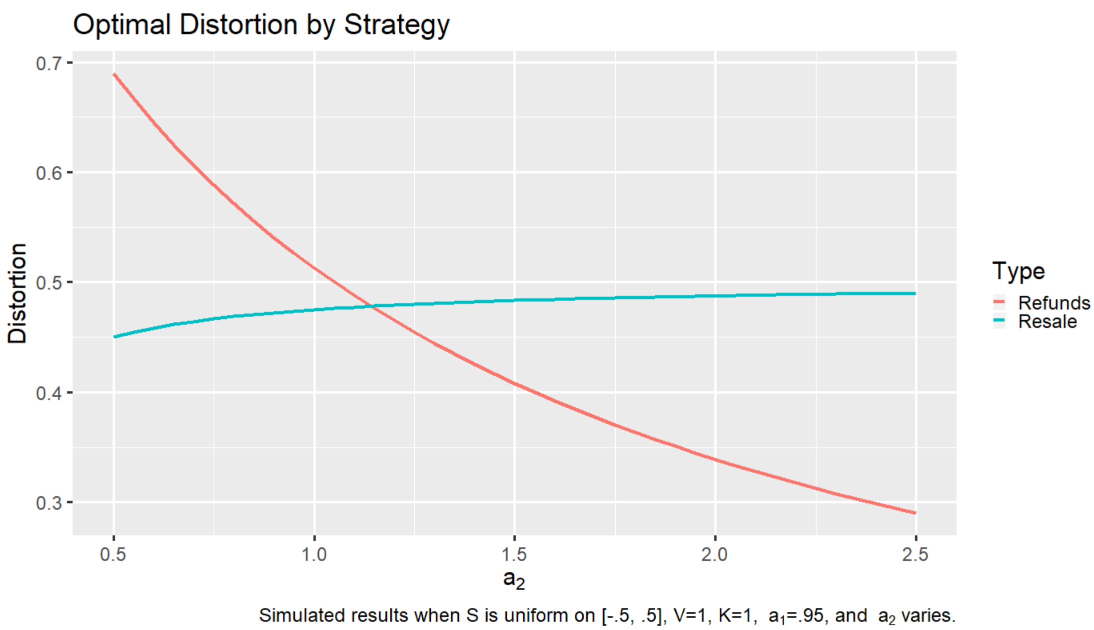


Figure 4: Optimal distortions for each strategy as a_2 varies.

significantly with a_2.

7 Conclusion

The goal of this paper has been to determine when sellers of event tickets should allow resale and when they should offer refunds. Resale is often derided but remains by far the most popular choice for ticket sellers. I contribute by developing a model that explains the use of resale when demand is uncertain and the seller has rigid prices.

The analysis demonstrates that resale offers deeper benefits. When the seller operates its own resale market, its profit cannot be higher with refunds. When it sells to brokers and owns the resale market, it is able to earn the same profit as if there were no rigidities and it could offer flexible refund contracts. The findings help explain why sports teams not only tolerate resale, but have developed partnerships or sponsorship deals with major resale platforms.

The conclusions of the study stretch beyond the market for tickets. The primary consideration—whether sellers have or need flexible prices—applies to sellers of perishable goods more broadly. It is unsurprising that professional sports teams, which face widely varying demand and have prices that are difficult to revise, prefer resale. Sellers of fashions, with short sales horizons and significant uncertainty over what will be popular, may also benefit from resale. But sellers with flexible prices, like hotels, would not benefit. For sellers with price rigidities but predictable demand, like organizers of major concert tours, resale is not necessarily the best option.

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A Materials for Web Appendix

For evidence that primary market prices are relatively inflexible, consider prices over time for each NFL team. Data come from Rodney Fort's Sports Business Data, a database consisting of ticket prices, team values, and other data on professional sports for all North American leagues.

Prices very rarely decrease and overwhelmingly follow a smooth upward trajectory. Teams do not appear to tailor prices to demand across seasons despite the fact that demand is widely variable—that is, the prices exhibit substantial rigidities. Occasional sharp increases are often explained by new stadiums.

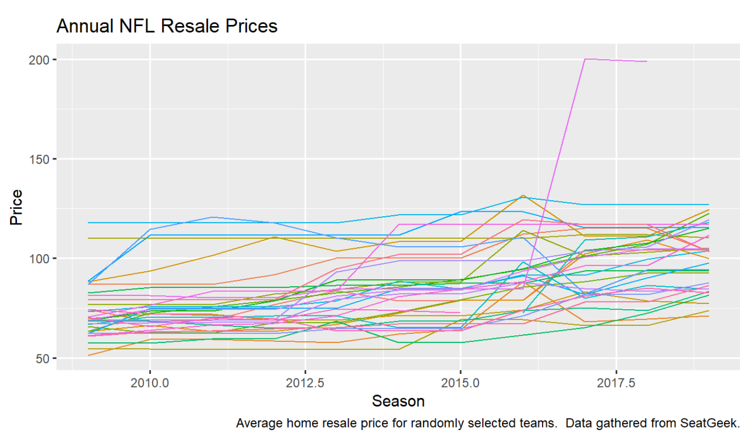

Figure 5: Weighted average primary market prices for all NFL teams.

B Proofs for Section 3 (Preliminaries and the Seller's Problem)

Proof. Let $\beta = \frac{a_1+a_2-K}{a_2}$. The seller's profit without reallocation is bounded above by

$$\pi^{NR} \leq a_1\Big(\alpha\int_{-V_L}^{\infty} V_L + s\, dG(s) + (1-\alpha)\int_{-V_H}^{\infty} V_H + s\, dG(s)\Big) + \\ (K-a_1)\big(\alpha(V_L + G^{-1}(\beta)) + (1-\alpha)(V_H + G^{-1}(\beta))\big).$$

The expression is an upper bound because it assumes the seller extracts all surplus from early arrivals and changes its price to $p_2 = V + G^{-1}(\beta)$ at each realization of V.

Profit is the same with third-party and integrated resale when there is no fee, so consider resale profit for both strategies. The seller's profit with resale is bounded below by the maximum of its resale profit when it sets $p_2 = V_L + G^{-1}(\beta)$ and $p_2 = V_H + G^{-1}(\beta)$. When it sets a high price, it earns at least

$$\pi^{SM} \geq a_1\big(\alpha(V_L + G^{-1}(\beta)) + (1-\alpha)(V_H + G^{-1}(\beta))\big) + (K-a_1)(1-\alpha)(V_H + G^{-1}(\beta)).$$

where the seller extracts at least $V_L + G^{-1}(\beta)$ from early arrivals in the low state because the resale price will be at least $V_L + G^{-1}(\beta)$ in the second period. When it sets a low price it earns at least

$$\pi^{SM} \geq\ a_1\Big(\alpha(V_L + G^{-1}(\beta)) + (1-\alpha)\Big(\int_{G^{-1}(\frac{a_1+a_2-K}{a_2})}^{\infty} V_L + G^{-1}(\beta)\, dG(s) + \\ \int_{-\infty}^{G^{-1}(\frac{a_1+a_2-K}{a_2})} V_H + G^{-1}(\beta)\, dG(s)\Big)\Big) + (K-a_1)(V_L + G^{-1}(\beta)),$$

where the resale price in period two is $V_H + G^{-1}(\beta)$ for the high realization even though consumers with $s > G^{-1}(\beta)$ are able to purchase for $V_L + G^{-1}(\beta)$ in the primary market.

Under Assumption 3, the maximum of the two resale profits exceeds the upper bound on profit without reallocation. $\square$

Appendix Lemma 1. *Suppose demand is certain and $g(s)$ is log concave. Then $\frac{\partial \pi^{SM}}{\partial p_2}$ is monotonically decreasing in p_2 for all $\tau \geq 0$.*

Proof. Suppose $g(s)$ is log concave so that $\frac{g'(s)}{g(s)}$ is weakly decreasing in s. For $\tau \geq 0$,

$$
\begin{aligned}
g'(s)/g(s) &\leq g'(s-\tau)/g(s-\tau) \\
a_1 g'(s)g(s-\tau) &\leq a_1 g'(s-\tau)g(s) \\
a_2 g(s)g'(s) + a_1 g'(s)g(s-\tau) &\leq a_2 g(s)g'(s) + a_1 g'(s-\tau)g(s) \\
g'(s)/g(s) &\leq \frac{a_2 g'(s) + a_1 g'(s-\tau)}{a_2 g(s) + a_1 g(s-\tau)}.
\end{aligned}
$$

The hazard rate condition and the second derivative of $\frac{1-G(s)}{g(s)}$ imply that $\frac{g'(s)}{g(s)} \geq \frac{-g(s)}{1-G(s)}$, so

$$
\begin{aligned}
\frac{a_2 g'(s) + a_1 g'(s-\tau)}{a_2 g(s) + a_1 g(s-\tau)} \geq g'(s)/g(s) &\geq \frac{-g(s)}{1-G(s)} \\
(a_2 g'(s) + a_1 g'(s-\tau))(1-G(s)) + (a_2 g(s) + a_1 g(s-\tau))g(s) &\geq 0,
\end{aligned}
$$

implying that $\frac{\partial}{\partial s}\frac{(a_1+a_2)(1-G(s))}{a_2 g(s)+a_1 g(s-\tau)} \leq 0$. Replacing s with $p_2 - V$ gives a reduced version of the seller's first order condition when $p_2 \geq p_2^r(p_2, V)$. The optimal price satisfies the condition because of efficient rationing. □

Appendix Lemma 2. *When demand is certain, the seller sells all of its inventory with resale, no reallocation, and no advance sales.*

Proof. The seller's optimal price to sell all inventory with resale is $p_2 = V + s_H(\tau)$. By Appendix Lemma 1, the first order condition is monotone in price under the assumption on $g'(s)$. Therefore the seller will sell all of its units if the first order condition is weakly negative at $V + s_H(\tau)$. At $p_2 = V + s_H(\tau)$, the condition is

$$
(a_1 + a_2)(1 - G(s_H(\tau))) - a_1 G(s_H(\tau) - \tau) - (V + s_H(\tau))(a_2 g(s_H(\tau)) + a_1 g(s_H(\tau) - \tau)) = 0.
$$

Solving for V gives the term in equation 7. Next consider profit and its first order condition with no reallocation,

$$
\pi^{NR}(p_2) = a_1 \left(\int_{-V}^{\infty} V + s \, dG(s) - \int_{p_2 - V}^{\infty} V + s - p_2 \, dG(s) \right) + p_2 \min\{a_2(1 - G(p_2 - V)), K - a_1\}
$$

$$\frac{\partial}{\partial p_2}\pi^{NR}(p_2) = (a_1+a_2)(1-G(p_2-V)) - a_2 p_2 g(p_2-V) = 0.$$

The first order condition is monotone in p_2 by Assumption 2 and is negative at the price exhausting all inventory, $p_2 = V + G^{-1}(\frac{K-a_1}{a_2})$, by the second term in the maximum statement.

Under the same condition, the seller sells all inventory with no advance sales. The first order condition without advance sales is

$$(a_1+a_2)(1-G(p_2-V)) - (a_1+a_2)p_2 g(p_2-V) = 0,$$

which is monotone in p_2 and always smaller than the first order condition without reallocation.

$\square$

C Proofs for Section 4 (Third-Party Resale and Refunds)

Lemma 2. *When demand is certain, profit is strictly higher with refunds than with resale.*

Proof. Suppose $\tau > 0$. By Appendix Lemma 2, the seller chooses the highest price with resale that exhausts inventory, $p_2^{SM} = V + s_H(\tau)$. The resale price will be $p_2^r(p_2^{SM}, V) = p_2^{SM}$. With refunds, the seller can set $p_2 = p_2^{SM}$ and $r = p_2^{SM} - \tau$. Each term in the profit expressions, equations (2) and (4), is the same except that resale profit is reduced by the amount paid in fees, $a_1 G(p_2^{SM} - \tau)\tau$. Refund profit is strictly higher.

If $\tau = 0$, profit is strictly higher because $r^{RC} < p_2^{RC}$. I show that $r = p_2$ is not optimal. The optimal prices with $r = p_2$ satisfy $(a_1+a_2)(1-G(V-r)) = K$. The first order condition at that menu is

$$\begin{aligned}
\frac{\partial \pi^{RC}}{\partial r} &= a_1\left(-rg(r-V) + \int_{p_2(r)-V}^{\infty} \frac{\partial p_2(r)}{\partial r}\, dG(s) + 0\right) + \\
&\quad \frac{\partial p_2(r)}{\partial r}(K - a_1(1-G(r-V))) + p_2(r) a_1 g(r-V) \\
&= a_1 \int_{r^0-V}^{\infty} \frac{\partial p_2(r)}{\partial r}\, dG(s) + \frac{\partial p_2(r)}{\partial r}(K - a_1(1-G(r-V))) < 0,
\end{aligned}$$

where $\frac{\partial p_2(r)}{\partial r} < 0$ because a firm receiving more refunded tickets must lower its price to sell more units in the second period. $\square$

Appendix Lemma 3. *When demand is certain, p_2^{RC} and r^{RC} increase linearly in the value V and the difference $p_2^{RC} - r^{RC}$ does not change with V.*

Proof. The seller sets p_2 to just exhaust its inventory for a given refund r. The first order condition for profit with respect to r is

$$\frac{\partial \pi^{RC}}{\partial r} = a_1\big(-rg(r-V) + \frac{\partial p_2(r)}{\partial r}(1-G(p_2(r)-V))\big) + (K - a_1 + a_1 G(r-V))\frac{\partial p_2(r)}{\partial r} + a_1 p_2(r) g(r-V).$$

Let p_2 and r be optimal at $V = v$, so they solve the first order condition and satisfy

$$K - a_1 + a_1 G(r-v) = a_2(1 - G(p_2(r) - v)). \tag{10}$$

Suppose the value changes to v' and consider candidate menu $p_2' = p_2 + v' - v$, $r' = r + v' - v$. The new menu satisfies equation 10 and thus $\frac{\partial p_2'(r')}{\partial r'} = \frac{\partial p_2(r)}{\partial r}$. The first order condition is

$$\begin{aligned}
&= a_1\big(-(r+v'-v)g(r+v'-v-v') + \frac{\partial p_2'(r')}{\partial r'}(1-G(p_2+v'-v-v'))\big) + \\
&\quad \frac{\partial p_2'(r')}{\partial r'}(K - a_1 + a_1G(r+v'-v-v')) + a_1 p_2 g(r+v'-v-v') \\
&= a_1\big(-(r+v'-v)g(r-v) + \frac{\partial p_2(r)}{\partial r}(1-G(p_2-v))\big) + \\
&\quad \frac{\partial p_2(r)}{\partial r}(K - a_1 + a_1 G(r-v)) + a_1(p_2+v'-v)g(r-v) = 0.
\end{aligned}$$

□

Appendix Lemma 4. *When demand is certain, the difference in refund and resale profit does not vary with V.*

Proof. Appendix Lemma 3 suggests that the seller's optimal refund menu is $r = V + \bar{s}_1$, $p_2 = V + \bar{s}_2$ for some constants $\bar{s}_1$, $\bar{s}_2$. Profit is

$$\begin{aligned}
\pi^{RC} &= a_1\left(\int_{\bar{s}_1}^{\infty} V + s\, dG(s) - \int_{\bar{s}_2}^{\infty} s - \bar{s}_2\right) + (V + \bar{s}_2)(K - a_1 + a_1 G(\bar{s}_1)) \\
&= KV + a_1\left(\int_{\bar{s}_1}^{\bar{s}_2} s\, dG(s) + \bar{s}_2(1 - G(\bar{s}_2))\right) + \bar{s}_2(K - a_1 + a_1 G(\bar{s}_1)).
\end{aligned}$$

At the optimal resale price $p_2 = V + s_H(\tau)$ with resale, profit is

$$
\begin{aligned}
\pi^{SM} = \;& a_1\Big(\int_{s_H(\tau)-\tau}^{\infty} V + s\,dG(s) + \int_{-\infty}^{s_H(\tau)-\tau} V + s_H(\tau) - \tau\,dG(s) - \\
& \int_{s_H(\tau)}^{\infty} V + s - V - s_H(\tau)\,dG(s)\Big) + (K - a_1)(V + s_H(\tau)) \\
&= KV + \left(s_H(\tau)(1 - G(s_H(\tau))) + \int_{s_H(\tau)-\tau}^{s_H(\tau)} s\,dG(s) + (s_H(\tau) - \tau)G(s_H(\tau) - \tau)\right) + \\
& (K - a_1)s_H(\tau).
\end{aligned}
$$

□

Proposition 1. *Refunds are more profitable than resale if*

1. $V_H - V_L \leq \underline{\Delta}$ *for some* $\underline{\Delta} > 0$*, or*
2. $\alpha \leq \underline{\alpha}$ *for some* $\underline{\alpha} > 0$*, or*
3. $\alpha \geq \bar{\alpha}$ *for some* $\bar{\alpha} < 1$.

Proof. 1. By Appendix Lemma 4, there is a constant difference $d \equiv \pi^{RC} - \pi^{SM}$ when $\alpha \in \{0, 1\}$. Suppose that the seller is able to set a different price p_2 in each state of the world with resale, earning $\pi^{SM,L}$ when V_L is realized and $\pi^{SM,H}$ otherwise, but sets its prices with refunds as if V_H were realized with certainty. The difference in profit is

$$
\begin{aligned}
\pi^{RC} - \pi^{SM} = \alpha\Big(& a_1\big[\int_{r^{RC}-V_L}^{\infty} V_L + s\,dG(s) - \int_{p_2^{RC}}^{\infty} V_L + s - p_2^{RC}\,dG(s)\big] + \\
& p_2^{RC}(K - a_1 + a_1 G(r^{RC} - V_L)) - \pi^{SM,L}\Big) + (1 - \alpha)d.
\end{aligned}
$$

For $\alpha < 1$, continuity implies that there exists $\underline{\Delta}$ such that for $V_L > V_H - \underline{\Delta}$ the expression is positive.

2. Consider the strategy above. For any V_L and V_H, there exists an α small enough that refunds are more profitable.

3. Suppose now that the seller still earns $\alpha\pi^{SM,L} + (1 - \alpha)\pi^{SM,H}$ with resale, but sets prices with refunds as if V_L were realized with certainty. The argument proceeds as before. □

Appendix Lemma 5. *For a random variable* X *with density* $f(x)$ *and finite expectation,* $\lim_{x\to\infty} x(1 - F(x + d)) = 0$ *for any constant* d.

Proof. Because

$$\lim_{u\to\infty} \mathrm{E}(X) - \int_{-\infty}^{u} xf(x)\,dx = \lim_{u\to\infty} \int_{u}^{\infty} xf(x)\,dx = 0$$

and $\int_u^\infty xf(x)\,dx \geq u\int_u^\infty f(x)\,dx = u(1-F(u))$, we have $\lim_{x\to\infty} x(1-F(x)) = 0$. Then

$$\lim_{x\to\infty} x(1-F(x+d)) + d(1-F(x+d)) = \lim_{x\to\infty} (x+d)(1-F(x+d)) = 0$$

and the fact that $\lim_{x\to\infty} d(1-F(x+d)) = 0$ delivers the result. $\square$

Proposition 2. *Let $\alpha \in (0,1)$ and define*

$$\begin{aligned} h(v) &= \alpha(V + s_L(\tau)) + (1-\alpha)(K(V + s_H(\tau)) - \pi^{RC}) \\ \underline{V}_L &= \inf\{V : h(v) > 0, \alpha(a_1 \int_{-V_L}^{\infty} V_L + s\,dG(s) - a_1 V_L < h(v)\}, \end{aligned}$$

where $s_L(\tau)$ solves $a_1 G(s_L(\tau) - \tau) = a_2(1 - G(s_L(\tau)))$.

For any $V_L > \max\{\underline{V}, \underline{V}_L\}$, there exists $\bar{V}_H(V_L)$ such that resale is more profitable than refunds when $V_H \geq \bar{V}_H(V_L)$ and $\tau = 0$. For each $V_L > \underline{V}_L$ and $V_H \geq \bar{V}_H(V_L)$, there exists $\bar{\tau}(V_L, V_H) > 0$ such that resale is more profitable when $\tau < \bar{\tau}$.

Proof. The first step is to establish an upper bound for profit with refunds. Let $V_H > V_L + 2d$ for some $d > G^{-1}(\frac{K}{a_1+a_2})$. Define p_2^H and p_2^L to be the seller's optimal prices with refunds when α is zero or one, and define $\pi^{RC,H}$ and $\pi^{RC,L}$ to be the analogous profits with refunds. By Appendix Lemma 3, the prices increase linearly in values V_H and V_L.

The optimal price is not known in general, so consider bounds for profit when the price and refund are set in different ranges. Note that $r \leq p_2$ without loss of generality. When $p_2 \leq p_2^L + d$,

$$\begin{aligned} \pi^{RC}(p_2, r) \leq\ & \alpha\pi^{RC,L} + (1-\alpha)\Big(a_1\{\int_{r-V_H}^{\infty} V_H + s\,dG(s) - \int_{s^{*RC}}^{\infty} V_H + s - p_2\,dG(s)\} + \\ & (K - a_1 + a_1 G(r - V_H))p_2\Big) \\ \leq\ & \alpha\pi^{RC,L} + (1-\alpha)\Big(a_1 \int_{-V_H}^{\infty} V_H + s\,dG(s) + (K - a_1 + a_1 G(p_2^L + d - V_H))(p_2^L + d)\Big) \equiv \pi_1^{RC}. \end{aligned}$$

Let $p_2^{\mathrm{NR},i} = V_i + G^{-1}(\frac{K-a_1}{a_2})$, the seller's optimal price at value V_i when it does not allow reallocation, and note that $p_2(1-G(p_2-V_i))$ is maximized at $p_2 = p_2^{\mathrm{NR},i}$. When $p_2 \geq p_2^H - d$ and $r > p_2^H - d$,

$$\begin{aligned}
\pi^{RC}(p_2, r) &\leq \alpha\Big(a_1\{\int_{r-V_L}^{\infty} V_L + s\,dG(s) - \int_{p_2-V_L}^{\infty} V_L + s - p_2\,dG(s)\} + a_2(1-G(p_2-V_L))p_2\Big) + \\
&\quad (1-\alpha)\pi^{RC,H} \\
&\leq \alpha\Big(a_1\int_{p_2^L+d-V_L}^{\infty} V_L + s\,dG(s) + a_2(1-G(p_2^H-d-V_L))(p_2^H-d)\Big) + (1-\alpha)\pi^{RC,H} \equiv \pi_2^{RC},
\end{aligned}$$

where $a_2(1-G(p_2^H-d-V_L))(p_2^H-d) > a_2(1-G(p_2-V_L))p_2$ because $p_2 > p_2^H - d > p_2^{\mathrm{NR},L}$ and the first order condition is monotone.

When $p_2 \geq p_2^H - d$ and $r \leq p_2^H - d$,

$$\begin{aligned}
\pi^{RC}(p_2, r) &\leq \alpha\Big(a_1\int_{-V_L}^{\infty} V_L + s\,dG(s) + a_2 \max_{p_2 \geq p_2^H - d}(1-G(p_2-V_L))p_2\Big) + \\
&\quad (1-\alpha)\Big(a_1\int_{p_2^H-d-V_H}^{\infty} V_H + s\,dG(s) + a_1 G(p_2^H-d-V_H)p_2^{\mathrm{NR},H} + (K-a_1)p_2^{\mathrm{NR},H}\Big) \\
&\leq \alpha\Big(a_1\int_{-V_L}^{\infty} V_L + s\,dG(s) + a_2(1-G(p_2^L+d-V_L))(p_2^L+d)\Big) + \\
&\quad (1-\alpha)\Big(a_1\int_{p_2^H-d-V_H}^{\infty} V_H + s\,dG(s) + a_1 G(p_2^H-d-V_H)p_2^{\mathrm{NR},H} + (K-a_1)p_2^{\mathrm{NR},H}\Big) \equiv \pi_3^{RC}.
\end{aligned}$$

Finally, for any price in $[p_2^L + d, p_2^H - d]$ and any refund, profit is

$$\begin{aligned}
\pi^{RC}(p_2, r) &\leq \alpha\Big(a_1\int_{-V_L}^{\infty} V_L + s\,dG(s) + \max_{p_2^L+d \leq p_2 \leq p_2^H - d} a_2(1-G(p_2-V_L))p_2\Big) + \\
&\quad (1-\alpha)\Big(a_1\int_{r-V_H}^{\infty} V_H + s\,dG(s) + a_1 G(p_2^H-d-V_H)p_2^{\mathrm{NR},H} + (K-a_1)(p_2^H-d)\Big) \\
&\leq \alpha\Big(a_1\int_{-V_L}^{\infty} V_L + s\,dG(s) + a_2(1-G(p_2^L+d-V_L))(p_2^L+d)\Big) + \\
&\quad (1-\alpha)\Big(a_1 V_H + a_1\int_{-\infty}^{p_2^H-d-V_H} p_2^H - d - V_H - s\,dG(s) + (K-a_1)(p_2^H-d)\Big) \equiv \pi_4^{RC}.
\end{aligned}$$

Next I show that profit with resale when $\tau = 0$ can exceed $\max\{\pi_1^{RC}, \pi_2^{RC}, \pi_3^{RC}, \pi_4^{RC}\}$. With resale, the seller earns at least as much as if it set $p_2 = V_H + s_H(\tau)$ and made no sales at V_L,

$$\pi^{SM} \geq \alpha a_1(V_L + s_L(\tau)) + (1-\alpha)K(V_H + s_H(\tau)),$$

where $s_L(\tau)$ satisfies resale demand when no consumers buy in the primary market, $a_1G(s_L(\tau)-\tau) = a_2(1-G(s_L(\tau))$. The difference between resale profit and each candidate upper bound is

$$
\begin{aligned}
\pi^{SM}-\pi_1^{RC} \geq\ & \alpha a_1(V_L+s_L(\tau))+(1-\alpha)K(V_H+s_H(\tau))-\alpha\pi^{RC,L}- \\
& (1-\alpha)\Big(a_1\int_{-V_H}^{\infty} V_H+s\,dG(s)+(K-a_1+a_1G(p_2^L+d-V_H))(p_2^L+d)\Big) \\
=\ & \underbrace{(1-\alpha)\Big[(K-a_1)V_H+Ks_H(\tau)-(K-a_1)(p_2^L+d)\Big]-\alpha(\pi^{RC,L}-a_1(V_L+s_L(\tau)))-}_{\psi_1} \\
& (1-\alpha)\Big(a_1\int_{-V_H}^{\infty} V_H+s\,dG(s)-a_1V_H+a_1G(p_2^L+d-V_H)(p_2^L+d)\Big), \\
\pi^{SM}-\pi_2^{RC} \geq\ & \underbrace{\alpha a_1(V_L+s_L(\tau))+(1-\alpha)(K(V_H+s_H(\tau))-\pi^{RC,H})}_{\psi_2}-\alpha\Big(a_1\int_{p_2^L+d-V_L}^{\infty} V_L+s\,dG(s)+ \\
& a_2(1-G(p_2^H-d-V_L))(p_2^H-d)\Big), \\
\pi^{SM}-\pi_3^{RC} \geq\ & \alpha a_1(V_L+s_L(\tau))+(1-\alpha)K(V_H+s_H(\tau))-\alpha\Big(a_1\int_{-V_L}^{\infty} V_L+s\,dG(s)+ \\
& a_2(1-G(p_2^L+d-V_L))(p_2^L+d)\Big)-(1-\alpha)\Big(a_1\int_{p_2^H-d-V_H}^{\infty} V_H+s\,dG(s)+ \\
& a_1G(p_2^H-d-V_H)p_2^{\mathrm{NR},H}+(K-a_1)p_2^{\mathrm{NR},H}\Big) \\
\geq\ & \underbrace{(1-\alpha)\Big(K(V_H+s_H(\tau))-a_1V_H-(K-a_1)p_2^{\mathrm{NR},H}\Big)}_{\psi_3}- \\
& (1-\alpha)\Big(a_1G(p_2^H-d-V_H)p_2^{\mathrm{NR},H}+a_1\int_{p_2^H-d-V_H}^{\infty} s\,dG(s)\Big)+ \\
& \alpha a_1s_L(\tau)-\alpha\big(a_1\int_{-V_L}^{\infty} V_L+s\,dG(s)-a_1V_L+a_2(1-G(p_2^L+d-V_L))(p_2^L+d)\big), \\
\pi^{SM}-\pi_4^{RC} \geq\ & \underbrace{(1-\alpha)\big((K-a_1)(V_H+s_H(\tau)-p_2^H+d)+a_1s_H(\tau)\big)}_{\psi_4}-\alpha\Big(a_1\int_{-V_L}^{\infty} V_L+s\,dG(s)-a_1V_L+ \\
& a_2(1-G(p_2^L+d-V_L))(p_2^L+d)-a_1s_L(\tau)\Big)-(1-\alpha)\Big(\int_{-\infty}^{p_2^H-d-V_H} p_2^H-d-V_H-s\,dG(s)\Big).
\end{aligned}
$$

Let $V_L > \underline{V}_L$, so

$$\tilde{\psi}_2 \equiv \psi_2-\alpha\big(a_1\int_{-V_L}^{\infty} V_L+s\,dG(s)-a_1V_L\big) > 0.$$

There exists $d_0 > G^{-1}(\frac{K}{a_1+a_2})$ such that $\psi_4 > \psi_2$ and

$$\tilde{\psi}_1 = (1-\alpha)\Big[(K-a_1)(d_0 - p_2^L) + Ks_H(\tau)\Big] - \alpha(\pi^{RC,L} - a_1(V_L + s_L(\tau))) > 0$$

and observe that $\psi_1 > \tilde{\psi}_1$ when $V_H > V_L + 2d_0$ and that ψ_1 is monotonically increasing in d. Assumption 3 implies that $\psi_3 > 0$ when $\tau = 0$. Let $\epsilon = \min\{\tilde{\psi}_1, \tilde{\psi}_2, \psi_3\}$. I show that all remaining terms can be made small.

Recall that $p_2^i - V_i$ is constant by Appendix Lemma 3; $p_2^{\mathrm{NR},i} - V_i$ is similarly constant. Fixing V_L at the value selected above, choose $d > d_0$ so that

$$\alpha a_1 \int_{p_2^L + d - V_L}^{\infty} V_L + s\, dG(s) < \epsilon/3 \qquad (\pi_2^{RC})$$

$$(1-\alpha)a_1 G(p_2^H - d - V_H) p_2^{NR,H} < \epsilon/5 \qquad (\pi_3^{RC})$$

$$(1-\alpha)a_2 \int_{p_2^H - d - V_H}^{\infty} s\, dG(s) < \epsilon/5 \qquad (\pi_3^{RC})$$

$$\alpha a_2 (1 - G(p_2^L + d - V_L))(p_2^L + d) < \epsilon/5 \qquad (\pi_3^{RC})$$

$$a_2 (1 - G(p_2^L + d - V_L))(p_2^L + d) < \epsilon/4 \qquad (\pi_4^{RC})$$

$$(1-\alpha)a_1 \int_{-\infty}^{p_2^H - d - V_H} p_2^H - V_H - s - d\, dG(s) < \epsilon/3. \qquad (\pi_4^{RC})$$

The fourth and fifth selections are possible because of Appendix Lemma 5. Given the d selected above, choose $V_H > V_L + 2d$ so that

$$(1-\alpha)\Big(a_1 \int_{-V_H}^{\infty} V_H + s\, dG(s) - a_1 V_H\Big) < \epsilon/2 \qquad (\pi_1^{RC})$$

$$(1-\alpha)a_1 G(p_2^L + d - V_H)(p_2^L + d) < \epsilon/2 \qquad (\pi_1^{RC})$$

$$\alpha a_2 (1 - G(p_2^H - d - V_L))(p_2^H - d) < \epsilon/3. \qquad (\pi_2^{RC})$$

The first selection follows from $\mathrm{E}(S) = 0$ and the third follows from Appendix Lemma 5. All differences $\pi^{SM} - \pi_i^{RC}$ are thus positive. ϵ is non-decreasing in V_H and the subtracted terms are non-increasing in V_H, so resale remains more profitable at higher values of V_H. The existence of $\bar{\tau}$ follows from the continuity of resale profit in τ. $\square$

D Proofs for Section 5 (Integrated Resale)

Proposition 3. *The seller earns weakly more when it owns and operates a resale market than when it offers a refund.*

Proof. Suppose the seller owns and operates the resale market and picks $p_2 = p_2^{RC}$, $\tau = p_2^{RC} - r^{RC}$. I show that the seller earns more at each realization of V.

First I show that the optimal refund price p_2^{RC} satisfies $p_2^r(p_2^{RC}, V_H) \geq p_2^{RC}$. Suppose not, implying $p_2^r(p_2^{RC}, V_L) < p_2^{RC}$ at V_L. The seller does not exhaust its inventory in either state, so Appendix Lemma 2 suggests that lowering p_2 from p_2^{RC} would increase profit in both states. Hence p_2^{RC} cannot be optimal. Similarly, $p_2^r(p_2^{RC}, V_L) \leq p_2^{RC}$. If not, there is rationing in both states and the seller can profitably raise p_2.

For any realization with resale price $p_2^r(p_2^{RC}, v) = p_2^{RC}$, equations (2) and (4) show that allocations and profit are the same with refunds and an owned resale market. When $p_2^r(p_2^{RC}, V_H) > p_2^{RC}$, fewer units are rationed and the seller can extract more surplus in the first period, strictly raising profit with an owned resale market. When $p_2^r(p_2^{RC}, V_L) < p_2^{RC}$, all sales at V_L take place at the resale price, which consumers anticipate in the first period. Profit is thus analogous to setting $p_2^r(p_2^{RC}, V_L)$ with the same fee, which increases profit by Appendix Lemma 2. $\square$

Proposition 4. *When the seller owns the resale market and sells to brokers in the first period, it earns as much as if it offered a menu of flexible prices and refunds, $\{p_2^{RC}(V), r^{RC}(V)\}$.*

Proof. Let $\{(p_2^{RC}(V_L), r^{RC}(V_L)), (p_2^{RC}(V_H), r^{RC}(V_H))\}$ be the firm's menu with flexible prices and refunds. By Appendix Lemma 3, $p_2^{RC}(V_L) - r^{RC}(V_L) = p_2^{RC}(V_H) - r^{RC}(V_H)$. Suppose the firm sets $\tau = p_2^{RC}(V_H) - r^{RC}(V_H)$ and sells its remaining $K - a_1$ units to brokers for $\mathrm{E}(p_2^{RC}(V)) - \tau$ per unit.

If a_1 units are sold to early arrivals, the resale price at V satisfies

$$K - a_1 + a_1 G(p_2^r(p_2, V) - \tau - V) = a_2(1 - G(p_2^r(p_2, V) - V)).$$

Substituting for τ and noting that the seller's menu with refunds clears the market at each V by Appendix Lemma 2, the only possible solution is $p_2^r(p_2, V) = p_2^{RC}(V)$. It follows that brokers are just willing to purchase at $\mathrm{E}(p_2^{RC}(V)) - \tau$.

Comparison of the profit equations (2) and (4) demonstrates that profit is the same: (i) the seller earns the same total revenue $p_2^{RC}(V)$ on the $K - a_1$ units sold in the second period, (ii) consumers expect the same prices $p_2^{RC}(V)$ in the second period, and (iii) the value thresholds for reallocating, $r^{RC}(V) = p_2^{RC}(V) - \tau$, are the same. □

Corollary 1. *Profit under resale with ownership and sales to brokers is strictly higher than with refunds when demand is uncertain, $\alpha \in (0, 1)$.*

Proof. By Appendix Lemma 3, the optimal menu with flexible prices and refunds is of the form $p_2^{RC}(V) = V + \bar{s}_1$, $r^{RC}(V) = V + \bar{s}_2$. The menu cannot be replicated with a single refund contract. The single refund must produce lower profit in at least one state. □

E Proofs for Section 6 (Welfare)

Lemma 3. *Total and consumer welfare only depend on the distortion δ. Both decrease in δ.*

Proof. Let $\tau = \delta$ and r satisfy $p_2(r) - r = \delta$. The seller wishes to sell all inventory with both strategies. For resale, $p_2(\tau)$ satisfies $K - a_1(1 - G(p_2(\tau) - \tau - v)) = a_2(1 - G(p_2(\tau) - v))$. For refunds, $p_2(r)$ satisfies $K - a_1(1 - G(r - v)) = a_2(1 - G(p_2(r) - v))$. The conditions are identical when $p_2 - r = \delta$, implying that $p_2(\tau) = p_2(r)$.

Total and consumer welfare can be written to only depend on p_2 and δ and are therefore the same for a common δ,

$$\begin{aligned} TW = \; & a_1 \int_{p_2 - \delta - v}^{\infty} v + s \, dG(s) + a_2 \int_{p_2 - v}^{\infty} v + s \, dG(s) \\ CW = \; & (a_1 + a_2) \int_{p_2 - v}^{\infty} v + s \, dG(s). \end{aligned}$$

□

Appendix Lemma 6. *Suppose that S has compact support $[\underline{s}, \bar{s}]$ and $g(s)$ is continuous. Let $r^{RC}(a_2)$ be the seller's optimal refund when there are a_2 arrivals in period two. There exists $\tilde{a}_2$ such that $\inf_{a_2 > \tilde{a}_2} r^{RC}(a_2) > v + \underline{s}$.*

Proof. The seller's first order condition with refunds and certain demand is

$$\frac{\partial \pi^{RC}}{\partial r} = a_1(p_2(r) - r)g(r - v) + \frac{\partial p_2(r)}{\partial r}(K + a_1 G(r - v) - a_1 G(p_2(r) - v)) = 0. \tag{11}$$

As a_2 grows, $p_2(r)$ approaches $v + \bar{s}$ and $\frac{\partial p_2(r)}{\partial r}$ approaches zero. Let $\epsilon > 0$ be small and select $\tilde{a}_2$ such that $v + \bar{s} - p_2(v + \underline{s}) < \epsilon$ and $|\frac{\partial p_2(v+\underline{s})}{\partial r}| < \frac{a_1}{K}(\bar{s} - \underline{s} - \epsilon)g(\underline{s})$ for all $a_2 > \tilde{a}_2$. It follows that the seller's first order condition is positive at $r = v + \underline{s}$ for $a_2 > \tilde{a}_2$. By continuity, the first order condition is also positive for $a_2 > \tilde{a}_2$ on $[v + \underline{s}, v + \underline{s} + \delta)$ for some $\delta > 0$. Therefore $r^{RC}(a_2) > v + \underline{s} + \delta$ for all $a_2 > \tilde{a}_2$. □

Proposition 5. *Suppose that S has compact support $[\underline{s}, \bar{s}]$ and $g(s)$ is continuous. There exists $\bar{a}_2$ such that total and consumer welfare are higher with refunds when $a_2 > \bar{a}_2$.*

Proof. Rewrite the resale market operator's problem in terms of the implied refund and take the derivative to obtain the first order condition,

$$\max_r a_1(p_2(r) - r)G(r - v) \tag{12}$$

$$a_1\left((\frac{\partial p_2(r)}{\partial r} - 1)G(r - v) + (p_2(r) - r)g(r - v)\right) = 0. \tag{13}$$

Let $\tilde{a}_2$ be such that $\tilde{r} \equiv \inf_{a_2 > \tilde{a}_2} r^{RC}(a_2) > v + \underline{s}$, which exists by Appendix Lemma 6. Note that the first order condition for the seller, given in equation 11, is larger if

$$-a_1 G(r - v) < \frac{\partial p_2(r)}{\partial r}\big(K - a_1 G(p_2(r) - v)\big). \tag{14}$$

Because $\frac{\partial p_2(r)}{\partial r}$ is a continuous function with limit zero as a_2 grows, there exists $\bar{a}_2 > \tilde{a}_2$ such that $0 > \frac{\partial p_2(r)}{\partial r} > a_1 G(\tilde{r} - v)/K$. For $a_2 > \bar{a}_2$ we have for all $r > \tilde{r}$

$$\frac{\partial p_2(r)}{\partial r}\big(K - a_1 G(p_2(r) - v)\big) \geq \frac{\partial p_2(r)}{\partial r} K > -a_1 G(r - v). \tag{15}$$

Because the first order condition for refunds is strictly greater above $\tilde{r}$ and the seller's optimal refund always exceeds $\tilde{r}$, the seller's optimal refund is higher than that set by the resale market. $\square$