The Cho-Kreps Intuitive Criterion

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The discussion and explanation of the Cho-Kreps intuitive criterion in the lecture notes is a bit brief. This handout revisits it in more depth.

Definition

Recall that the Cho-Kreps intuitive criterion is used to evaluated PBEs in signaling games. A PBE *fails* the Cho-Kreps intuitive criterion if both:

1. Some type θ of player 1 can strictly profit by deviating (relative to PBE payoffs) if other players believe it to be type θ under the deviation.
2. No other type θ' can strictly profit (relative to the PBE) under any beliefs by making the *same* deviation as type θ.

Hopefully, part 1 of the definition is clear. Most of the confusion is with part 2. The two key points of part 2 are that θ' considers the same deviation that type θ finds profitable, and that we should evaluate the θ' deviation under the most favorable beliefs possible.

Intuition

The definition is rather abstract. Intuitively, what does it mean? The big idea is that the Cho-Kreps criterion rules out PBEs that rely on unrealistic beliefs. For example, the pooling PBE of the Spence model in the problem set assumes that $\mu(\theta_i|e) = \frac{1}{3}$ for all types θ_i and all levels of education. But, if any agent deviated to a higher level of education, we would expect the deviant to be the high type because he faces lower costs of education. In this sense, the beliefs are unrealistic—they are only used to stop the high type from deviating.

Now, let's connect the intuition to the definition. Suppose a PBE fails the criterion. From part 1, we know that some type would deviate if it were believed, so the beliefs, specifically the belief that assigns probability strictly less than one to type θ under the deviation, are needed to support the PBE (as in the pooling example of the education model).

So, why is a deviant not recognized as type θ? Not recognizing the deviant as type θ is only justifiable if another type might plausibly make the same deviation under favorable beliefs. If there is no plausible other type (i.e. part 2 is satisfied), then we can declare that the beliefs supporting the PBE are unreasonable and the PBE fails the Cho-Kreps criterion.

Applications

Spence Education Model

The only separating PBE satisfying the Cho-Kreps criterion is the least-cost PBE. Consider any separating PBE where the high type earns some level of education $e' > \underline{e}$, where $\underline{e}$ is the minimum level of education needed to be recognized as type θ_H. Then the high type is strictly better off by deviating to $\underline{e}$ if firms believe it to be the high type (satisfying part 1). From the calculations defining $\underline{e}$, the low type cannot strictly profit from the same deviation to $\underline{e}$ even under the most favorable belief, which is believing that any agents with $\underline{e}$ is the high type (satisfying part 2). Therefore only the least-cost PBE satisfies Cho-Kreps.

Beer-Quiche Game

Pooling on q PBE fails the criterion. A tough player 1 can strictly profit by deviating to b if she is recognized as the tough type (part 1). But a weak player 1 cannot strictly profit from the same deviation even if she is never challenged to a fight, implying the most favorable beliefs, (part 2): a weak player earns 2 when pooling on b instead of 3 when pooling on q.

The PBE with pooling on b satisfies Cho-Kreps because there is no type θ satisfying part 1 of the definition. The strong type does not want to deviate because it already earns its highest possible payoff. If the weak type deviated to q and were believed, player 2 would challenge her to a fight and she would earn $2 < 3$.